POWER LOAD SHEDDING SIMULATION AND OPTIMIZATION

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Abstract- A static methodology that optimizes load shedding to achieve a balance between generation and demand in an electrical power system is developed. The objective, under a power deficit in an emergency condition, is to reduce the non-served demand to its minimum. The load shedding problem is formulated as a non-linear optimization model that is solved through the Damped Lagrangian Penalty algorithm. Load models at each bus consider voltage-frequency characteristics and power factor. In the simulation of the problem, both active and reactive power and frequency are considered, thus achieving an integral optimization. The models developed are evaluated with a sample 8 bus system and are also applied to the 94 bus longitudinal Chilean Central Interconnected System. The proposed methodology demonstrates adequate for the planning and design of optimal load shedding schemes.

I INTRODUCTION

An electrical power system is subject to disturbances and faults that generally cause unexpected outages in the equipment and alter the set point of the system, thus degrading the electrical service. Severe disturbances take the system to an emergency status, making it necessary to implement control actions to prevent cascading outages in the equipment that would bring the collapse and disintegration of the system. Disturbances cause two types of emergencies called stability and viability crises.

The likelihood of having a generation deficit is very high and its effects are very severe. As generators are the most important and most costly equipment in the system, they are equipped with protection devices that automatically act upon the presence of a disturbance endangering the system. Thus, a power generation deficit is produced.

There are two options to face generation deficits:

i) Generation redispatch: the deficit is covered by the remaining generators, redispatching generation. This is only applicable in a system with enough spinning reserve.

ii) Load shedding: the deficit is balanced through load shedding in some buses of the system, setting frequency steps to shed load blocks in specific buses. Less important loads are shed when the system's frequency drops to a determined level according to a previously assigned priority list. This option can be applied when facing a viability crisis.

When designing a load shedding scheme, the common practice is to determine the load to be shed by means of transient stability studies. This shedding can be excessive, as the optimization of the shed load is not considered.

Various researchers have proposed solutions to achieve an optimal load shedding.

The article presented by Hajdu et al (1968) is one of the first works reported in literature to minimize load shedding. It proposes a computer procedure to minimize load reduction through the gradient technique, that is based on the Newton-Raphson technique to solve the power flow equations and on the Kuhn-Tucker theorem for optimization. However, the analysis does not consider reactive power nor frequency variations.

Subramanian (1971) describes a sensitivity model that through linear programming, solves the problem of optimal load shedding. The objective function is the quadratic variation of the load current. The model does not consider the operational requirements nor the frequency of the system.

Chan and Schweppe (1979) consider the problem of re dispatching generation and load shedding under emergencies. A non-linear optimization model is presented. Its resolution is obtained through an exact sensitivity model. The objective function is the summation of the non-linear penalization of the load shedding and the deviations in the generation dispatch. Due to the size of the problem, the technique of sparse linear programming with an upper limit is used, thus exploiting the sparsity of the complete linear formulation. However, it doesn't consider the system's frequency and it can only be applied to eliminate overloads in the equipment.

Palaniswamy et al (1985) develop a method for optimal load shedding in a power system. The modeling considers the dependence of the load on voltage and frequency, together with the effect of the generator controls and voltage regulators. The objective function is the frequency and voltage dependent load shedding, minimizing load curtailment as well as the frequency deviation from the nominal value, subject to operational and network constraints. The model is solved as two sub-problems, one considering active power and frequency and the other one considering reactive power and voltages. It is solved iteratively until the maximum value of the objective function is smaller than a specified error. The proposed modeling does not deliver a net load shedding and the power balance is obtained through the load and generation variations with frequency, which can be insufficient.

Medicherla et al (1979) analyze the relief of overloads in lines and transformers in an electrical power system through the redispatching of generation and load shedding, considering sensitivity coefficients. The model is composed by two sets of equations; the linear relationship between the overload currents and the voltages, the state variables and the generated power. The matrix relationship of overload currents and the state variables result in a matrix, that is rectangular and highly sparse. The pseudo-inverse of that matrix is considered, which has not a single solution.

Kuppurajulu (1985) presents an algorithm for emergency control to be incorporated as an important function in a power control center. The changes in generation and load are minimized. The model considers the effect of the speed regulator and the setting of the automatic generation control. It determines the new post-contingency status and it eliminates overloads through a linear programming procedure. It uses the technique of linear power flows, using the model's flexibility, as it is not necessary to again invert the susceptance matrix for a specific contingency and/or later equipment outage.

Adibi and Thorne (1988) consider a strategic local load shedding, as the control actions to eliminate overloads in equipment are not sufficient (voltage regulation, transformer tap changes, start up of gas turbines). It's a heuristic method that considers a specified priority list on the load to be shed. It is based on the redistribution of the flows and a decrease in the load. Only the active power is considered and the load is modeled as constant power.

The optimization models analyzed for load shedding correspond to static models. The approaches used by the different researchers for the solution of the proposed models can be classified in: sensitivity coefficients analysis models (Subramanian, 1971; Adibi and Thorne, 1988; Price et al, 1988), and linear (Subramanian, 1971; Kuppurajulu, 1985) and non-linear (Palaniswamy et al, 1985) optimization models.

In the proposed methodology, the load is modeled considering its voltage-frequency characteristic, reflecting the dampening character of the load. The fact that the power factor stays constant during the shedding drives to a result that agrees with reality, simplifies modeling and reduces the amount of variables. As it considers that the modulus of the load is variable, it makes it possible to obtain the solution even under situations of extreme generation losses, because the load variation with voltage and frequency is of small magnitude and is not able to balance the deficit.

II Load modeling

In the electrical power system analysis, the load has been modeled as constant impedance in stability problems and as constant power in the calculation of power flows. The load modeling as a function of voltage and frequency has only been made in stability studies. Considering that the load is dependent on voltage, Berg (1972) proposes an exponential modeling. Price et al (1988) and Vaahedi et al (1988) postulate a polynomial type complete load model that considers the dependence of the load as a function of voltage and frequency.

It would be better to use a polynomial load model, but the determination of its parameters would only be valid for a specific system. Instead, the consideration that the load is composed by parts that behave as constant power, current and impedance is a natural extension that combines the known models of the load. In this work, the following dependence of the load on voltage and frequency is used:

$$P_{Di} = \underline{P}_{Di}\,(1+K_{pi}\Delta f)\,\{P_{pi}+P_{ci}(V_i/V_{io}) + P_{zi}(V_i/V_{io})^2\}$$

$$Q_{Di} = \underline{Q}_{Di}(1+K_{qi}\Delta f)\,\{Q_{pi}+Q_{ci}(V_i/V_{io}) + Q_{zi}(V_i/V_{io})^2\}$$

where:

$\underline{P}_{Di}, \underline{Q}_{Di}$	:	Modulus of active and reactive power load in bus i, in p.u., that do not depend on voltage and frequency
V_i	:	Modulus of voltage in bus i
V_{io}	:	Modulus of nominal voltage in bus i
Δf	:	Frequency deviation
K_{pi}, K_{qi}	:	Frequency constants
P_{pi}, Q_{pi}	:	Portion of the load modeled as constant power
P_{pi}, Q_{ci}	:	Portion of the load modeled as constant current
P_{zi}, Q_{zi}	:	Portion of the load modeled as constant impedance

Considering that in the load shedding the power factor is maintained constant, simplifies the modeling. There are less variables, because $Q_{Di} = P_{Di}\,\mathrm{tg}\,\theta_i$, where $\cos\theta$ is the power factor.

III Optimization model

In the process of optimization of the electrical power system operation under emergency conditions due to generation loss, the model is expressed as:

Minimization of non-served energy and service degradation subject to: network constraints and operation constraints.

The equivalent problem is translated into minimizing load shedding and frequency deviations. The complete model of the load shedding problem is expressed as:

$$\text{Min } \Sigma_i [\alpha_i (P_{Di} - P_{Dio})^2 + \beta_i (Q_{Di} - Q_{Dio})^2] + \gamma (f - f_o)^2$$

subject to

$$P_{Gi} - P_{Di} - P_i = 0$$
$$Q_{Gi} - Q_{Di} - Q_i = 0$$
$$P_{Gi} - P_{Ri} \Delta f / R_i = P_{Gio}$$
$$|\delta_i - \delta_j|^2 - \Psi_{ij}^2 \le 0$$
$$V_i^m \le V_i \le V_i^M$$
$$\Delta f^m \le \Delta f \le \Delta f^M$$
$$P_{Gi}^m \le P_{Gi} \le P_{Gi}^M$$
$$Q_{Gi}^m \le Q_{Gi} \le Q_{Gi}^M$$
$$Q_{Ci}^m \le Q_{Ci} \le Q_{Ci}^M$$
$$P_{Di}^m \le P_{Di} \le P_{Di}^M$$
$$Q_{Di}^m \le Q_{Di} \le Q_{Di}^M$$

where

$$P_i = V_i \Sigma_j V_j \{ G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \}$$
$$Q_i = V_i \Sigma_j V_j \{ G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \}$$

$G_{ij}+jB_{ij}$	:	Elements of the nodal admittance matrix
V_i, δ_i	:	Modulus and angle of the voltage in bus i
P_{Di}, Q_{Di}	:	Active and reactive power load in bus i
P_{Gi}, Q_{Gi}	:	Active and reactive power generation in bus i
P_{Gio}	:	Active power generation specified in bus i
Q_{Ci}	:	Reactive power at compensator in bus i
$\Delta f = f - f_o$	:	Frequency deviation
Ψ_{ij}	:	Maximum angular difference
P_{Ri}	:	Nominal power of generator i
R_i	:	Speed regulation of generator i
P_{Dio}, Q_{Dio}	:	Active and reactive power load in bus i, before the emergency
α_i, β_i	:	Weight penalty factors for active and reactive power load at bus i
γ	:	Weight penalty factor for frequency
m, M	:	Superscripts denoting lower and upper limits for the variables

The calculations are made with values in per unit and angles are expressed in radians. In the optimization model considered, the variables used are the following:

$$x = [\ V_i\ \ \delta_j\ \ \Delta f\ \ P_{Gi}\ \ Q_{Gi}\ \ Q_{Ci}\ \ P_{Di}\ \ Q_{Di}\ ]$$

IV Optimization Methodology

The optimal load shedding model can be presented as an optimization problem expressed in the following manner:

Min $f(x)$

subject to

$$g_i(x) = 0 \qquad i = 1, \dots, NE$$
$$g_i(x) \le 0 \qquad i = NE+1, \dots, N$$
$$a_j^t x = b_j \qquad j = 1, \dots, ME$$
$$a_j^t x \le b_j \qquad j = ME+1, \dots, M$$
$$l \le x \le u$$

where

$g_i(x)$	:	Non linear constraints
a_j^t, b_j	:	Coefficients of linear constraints
l, u	:	Lower and upper limits
NE	:	Number of equality non-linear constraints
N	:	Total number of non-linear constraints
ME	:	Number of equality linear constraints
M	:	Total number of linear constraints

It is a non-linear optimization model that is solved through the Damped Lagrangian Penalty algorithm as indicated in Contesse and Villavicencio (1989 and 1991), consisting on solving various problems of the type:

$$\text{Min } L[\ x, \lambda^k, \mu^k, r^k, \varepsilon^k, \rho^k\ ]$$

subject to

$$a_j^t x = b_j \qquad j = 1, \dots, ME$$
$$a_j^t x \le b_j \qquad j = ME+1, \dots, M$$

where

$$L[x, \lambda^k, \mu^k, r^k, \varepsilon^k, \rho^k] = L[x, \lambda^k] +$$

$$\frac{r^k}{2} \sum_{i=1}^{NE} g_i(x)^2 + \sum_{i=NE+1}^{N} \theta[g_i(x), \mu^k, r^k, \varepsilon^k, \rho^k]$$

where $L[x, \lambda^k] = f(x) + \Sigma \lambda_i^k g_i(x)$ is the classic Lagrangian function and $\theta(s, \mu^k, r^k, \varepsilon^k, \rho^k)$ is a penalty piecewise function. In each iteration, there is a convenient updating of the values of the variables. The method is a variant of the Hestenes-Powell-Rockafellar multipliers method (Contesse and Villavicencio, 1989). The damped penalty algorithm has the advantage that it does not calculate extraneous stationary points, keeping all its potential under favorable conditions.

V APPLICATIONS

To validate the methodology, the algorithm and the programs developed, simulations and comparison of results were made for standard 30 and 57 bus IEEE systems (Blanco, 1992). Results of studies of an eight bus sample system are reported in this work. Studies of the Chilean Central Interconnected System (CIS) are also illustrated.

The values considered for the constants in the load modeling are: $P_p=Q_q=0.5$, $P_c=Q_c=0.2$, $P_z=Q_z=0.3$, $K_p=K_q=0.05$

The eight bus sample system of the figure has 5 PV buses and 3 PQ ones. It has 8 lines and 2 transformers with adjustable taps.

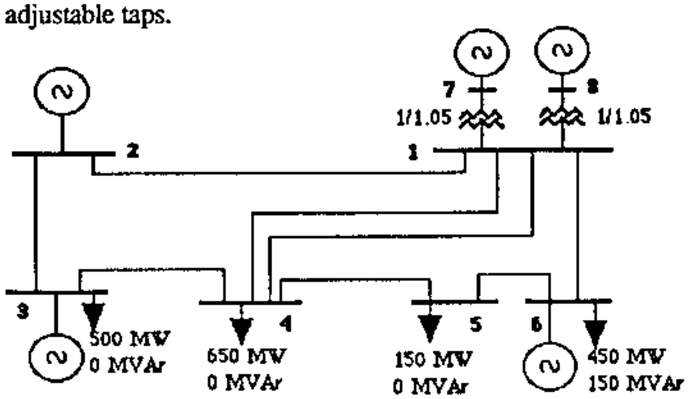

Table 1.-Line and transformer parameters (in p.u.,1000 MVA base, 50 Hz)

Lines		R	X	B/2
1	2	0.100	0.65	0.013
2	3	0.104	0.65	0.013
3	4	0.030	0.10	
4	5	0.030	0.20	
5	6	0.030	0.10	
1	6	0.104	0.65	0.013
1	4	0.104	0.65	0.013
1	4	0.104	0.65	0.013
Transformers				
1	7		0.20	
1	8		0.20	

The generation deficit considered is 800 MW (20.28% with respect to the total generation of the pre-emergency state), with a generation loss of 400 MW for bus 7 ($\Delta P_{Gi}/P_{Gio}$=48.73 %) and 400 MW for bus 6 ($\Delta P_{Gi}/P_{Gio}$=24.62 %). The results of the application are shown in Table 2 (the computer used was an Apollo HP 10000). The results obtained coincide significantly with those of Palaniswamy et al (1985). It would be necessary to know the modeling of the generation used in that reference to achieve a more accurate comparison.

Table 2.-Eight bus system- Summary of optimization results

Model used		P_D=const.	$P_D=P(V,f)$	P_D, Q_D
Objective function		1.37e-1	6.292e-1	6.298e-1
Frequency	Hz	50	49.77	49.77
Max. error	MW	-0.14	-0.048	-0.048
Gen. loss.	MW	800	800	800
Load shed	MW	703.92	501.34	501.34
	MVAr	53.54	39.60	0
Losses	MW	127.46	145.38	145.38
	MVAr	689.36	816.21	816.20
Computer requirements				
CPU time	s	63.5	37.8	286.2
Iterations		45	30	115

The CIS is modeled with 94 buses, 13 generation buses, 3 compensation buses, 103 lines, 34 transformers and 13 fixed shunt compensators. The main features of the CIS are its scarce meshing, as a longitudinal system, and the high sparsity of its nodal admittance matrix (96.54 %). The algorithm developed incorporates efficient computer sparsity programming (Blanco, 1992). The CIS often presents ill conditions in load flow studies.

The system studied has a total load of 2456 MW. The level of generation loss is 150 MW for bus Rapel13.8 ($\Delta P_{Gi}/P_{Gio}$=46.88 %) and 100 MW for bus Colbun13.8 ($\Delta P_{Gi}/P_{Gio}$=22.73 %).

Table 3.-CIS-94 bus system- Summary of optimization results

Model used		P_D=const.	$P_D=P(V,f)$
Objective function		0.350498	0.024988
Frequency	Hz	50	49.8
Max. error		0.0014 MVAr	-0.0027 MW
Gen. loss	MW	250	250
Load shed	MW	245.34	76.80
	MVAr	68.92	30.03
Losses	MW	124.66	132.57
	MVAr	197.80	360.54
Computer requirements			
CPU time,	s	6807.6	2046.8
Iterations		18	7

The results of the application of the algorithm to the CIS are shown in Table 3. It is possible to observe that the load modeling, which does not take into account its voltage-frequency characteristic, produces an excessive load shedding (98.1 %), if compared with the model that considers its voltage-frequency dependence (30.7 %).

Within the optimization process, the magnitude of the dual variables (Lagrange multipliers) indicates up to what point the minimization of the load shedding can still be improved. Tables 4 and 5 indicate the variables that are saturated (constraints with active limits) in the application of both models to the CIS and likewise, the associated Lagrange multipliers. When relaxing the limit constraints that are saturated, the resulting load shedding will be less. The technical conditions and the availability of resources will allow to relax those constraints.

Table 4.-CIS- Saturated variables. Model P_D=constant

Variables	Bus	Min.	Max.	λ(pu)
Voltage p.u.	JAHUEL110		1.08	3.3167
	ISLA154		1.08	12.9941
	VALDI220		1.08	1.6250
Compensation MVAr	MAINT13.8	-40.0		0.3778
Load MW	RAPEL66B	0.0		0.2616
	RAPEL66R	0.0		0.0753
	TILC154	0.0		0.0514
	PEDRO110		31.2	0.2610

Table 5.-CIS-Saturated variables. Model P_D=P(V,f)

Variables	Bus	Min.	Max.	λ(pu)
Frequency Hz		49.8		23.3967
Load MW	RAPEL66B	0.0		0.0174
	TILC1542	0.0		0.0427

From the analysis of the results, if the voltage-frequency characteristic of the load is not taken into account, the model is simple, however it needs more iterations due to the saturation of the limit constraints, thus making it more difficult to search for the solution. Also, there is an excessive load shedding, approximately of the same order as the generation deficit. Instead, when considering the voltage-frequency characteristic of the load, the power deficit is partly absorbed by the load variation with voltage and frequency; few limit constraints are saturated and the shed load is less for an equal power deficit. On the other hand, considering the active and reactive power of the load as independent variables implies results with no reactive power shedding, and that is not in accordance with reality; it also implies increasing the number of variables of the problem and the solution times.

VI Conclusions and further developments

A presentation is made of the general formulation of the optimal load shedding problem in electrical power systems, in order to face a generation deficit under an emergency condition.

Non linear models and optimization algorithms are proposed and developed. Successful applications to standard systems and a real network were achieved. Robust numerical results demonstrate them to be attractive to be used in the simulation, planning and design of optimal load shedding schemes.

As should be expected, the initial starting point (solution of the initial condition) in an optimal load shedding analysis demonstrates important for a fast convergence to the optimal solution (Blanco, 1992).

Not taking into account the dependence between the active and reactive powers of the load implies results that do not agree with real systems. Likewise, if in the load modeling there is no consideration given to the voltage-frequency characteristic, there will be an excessive load shedding, although resulting in a simple model with less variables.

The models developed in this work consider continuous variables for the load shedding. In practice, the action of the frequency relays produce a discrete load shedding. It would be convenient to develop optimization methodologies with continuous and discrete or mixed variables.

The model of the load used in this work considers that the load is composed of parts that are modeled as constant power, current and impedance and a linear dependence on frequency, both for active and reactive power. It would be desirable to make a more complete, detailed and accurate modeling of the loads in each of the CIS buses to adequately asses their behavior.

When considering the upper limits of flow in lines and transformers there are non-linear constraints that could unfold into linear constraints. That would increase the number of constraints, however, they have a very sparse structure that could be treated with advantage.

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