



## RECONFIGURATION OF ELECTRIC DISTRIBUTION SYSTEMS WITH A SIMPLIFIED POWER SUMMATION METHOD

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**ABSTRACT** A methodology to reconfigure an electric power distribution network under normal operating conditions to reduce the active losses of the network or to balance the load of the system's feeders is presented. An heuristic solution algorithm is used. It is based on the method of branch exchange, where different radial configurations are generated, improving the objective function and originating a sequence of maneuvers to be performed on the network. In order to guide the search for configurations, the Power Summation Method is used for load flow calculations, in iterative and direct simplified versions. Significant reductions in cpu time are achieved with the simplified version. Turbo Pascal programming facilitates the dynamic handling of the network structure.

**Keywords:** Distribution systems, Electric losses, Network reconfiguration, Load flow.

### 1. Introduction

Between 30 and 40 % of total investments in the electrical sector goes to distribution systems, but nevertheless, they have not received the technological impact in the same manner as the generation and transmission systems. Many of the distribution networks work with minimum monitoring systems, mainly with local and manual control of capacitors, sectionalizing switches and voltage regulators; and without adequate computation support for the system's operators. Nevertheless, there is an increasing trend to automate distribution systems to improve their reliability, efficiency and service quality. Automation is possible due to the advance microprocessor control technology, to its increasing cost reduction and due to its joint use with telecommunications technologies. It is possible to install distribution operation centers where the network is constantly monitored and control actions can be made remotely. With the aid of these technologies it is possible to monitor substations and feeders to reconfigure feeders and to control voltage and reactive power.

If the network reconfiguration and voltage control and reactive power adjustments become routine operations, the operators will not trust only on their criteria and experience to operate the system. It will be necessary to have dedicated software that assists the operator in selecting appropriate control actions. One of these actions is the network reconfiguration that can be oriented to different objectives. Under normal operating conditions, the network is reconfigured to reduce the system's losses and/or to balance load in the feeders. Under conditions of permanent failure, the network is reconfigured to restore the service, minimizing the zones without power.

Different algorithms have been used to solve the reconfiguration problem: combinatorial optimization with discrete branch and bound methods [1,10], expert system techniques [9,11] and heuristic methods [1-5,8]. One of the first works reported to reduce losses in a distribution network was presented by Merlin and Back [1]. It presents an integer-mixed non-linear optimization model that is solved through the discrete branch and bound method. Due to the combinatorial nature of the problem, it requires checking a great number of configurations for a real-sized system.

Shirmohammadi and Hong [2] use the same heuristic procedure exposed in [1], they share its advantages and prevent its main disadvantages. Civanlar et al [3] present a simple heuristic methodology to reduce network losses. They formulate a simple algebraic expression to estimate the active loss reduction due to

the load transfer between a pair of feeders. Castro et al [4] propose search heuristic techniques to restore the service and load balance of the feeders. Castro and Franca [5] propose modified heuristic algorithms to restore the service and load balance. The operation constraints are checked through a load flow solved by means of modified fast decoupled Newton-Raphson, artificially increasing the X/R ratio and applying an adequate transformation. Aoki et al [6-7] propose heuristic algorithms to restore the service and balance the load in the feeders. Baran and Wu [8] present a heuristic reconfiguration methodology based on the method of branch exchange to reduce losses and balance loads in the feeders. To assist in the search, two approximated load flows for radial networks with different degrees of accuracy are used. Also they propose an algebraic expression that allows to estimate the loss reduction for a given topological change. Liu et al [10] propose an expert system to solve the problem of restoration and loss reduction in distribution systems. The knowledge base is built through Prolog language for the restoration problem and in Pascal for loss reduction.

The model for the reconfiguration problem is a combinatorial non-linear optimization problem, because to find the optimal solution, it is necessary to consider all the possible trees generated due to the opening and closing of the switches existing in the network. If it is intended to determine an optimal solution, a method of discrete optimization can be used. Nevertheless, the time in computational resources is too high and thus, impractical. On the other hand, methods based on heuristic techniques allow to find a viable solution with a limited requirement of cpu time, so they are more adequate to be used in "on-line" processes. In general, these methods converge to a local optimum; no convergence to a global optimum is guaranteed.

In this work, the problem of reconfiguration of the distribution network under normal operation to reduce active losses and to balance loads in the system will be considered. The network reconfiguration consists on modifying the topology of the system by switching remotely controlled sectionalizing switches. In this process, the nodes can be energized through different paths through the interconnection with other feeders (substations) and/or interconnection of nodes belonging to a same feeder. Usually, distribution systems operate with a radial topological structure; consequently, the opening and closing of sectionalizing switches must be made considering this constraint. The problem consists on determining an ordered switching list that allows to reduce losses or to balance the load of the system without infringing operational and topological constraints.

This work modifies the solution methodology proposed by Baran and Wu [8]. To obtain a solution, that methodology requires to fulfill a large amount of load flow calculations, and due to the great computational effort involved, for a real sized distribution system it results impractical. As a solution to this difficulty, in this work we propose a simplified non-iterative calculation method that allows to calculate the power flows and the voltages of the buses of the system with reasonable accuracy, drastically reducing the computational effort. This simplified calculation method has been named "Simplified Power Summation Method", as it is presented as a natural consequence of the Power Summation Method [12], applicable to a radial network with lateral branches.

A computer program was developed using Turbo Pascal 5.5 The radial topological structure of the network is built dynamically using pointer variables. It was preferred to represent the system tree through a binary tree.

## 2. Problem formulation

### 2.1. Mathematical model

Given a radial distribution network with  $n$  nodes with a known topological structure, the problem consists on finding an optimal radial network  $t^*$  among all possible radial networks  $t_j$  generated with the switch condition changes, that minimizes the objective function and that does not infringe the network's load flows and operational constraints. The mathematical model can be expressed as:

$$\min C(x,t_i) \quad (1)$$

subject to:

$$F(x,t_i) = 0 \quad (2)$$

$$G(x,t_i) \leq 0 \quad (3)$$

where  $C(x,t_i)$  is the objective function to be minimized;  $F(x,t_i)$  is the vector of equality constraints and represents the load flow equations;  $G(x,t_i)$  is the vector of inequality constraints and corresponds to

operational constraints for the network;  $x=(P,Q,V)$  where  $P$  and  $Q$  represent the active and reactive powers of the receiving end of the branches of the network and  $V$  corresponds to the magnitudes of the voltages of the system's nodes. This is a combinatorial non-linear optimization problem.

## 2.2. Objective function and restrictions

The objective function to be minimized for the problem of active losses reduction in the system consists on the total active loss of the network and is expressed as [8]:

$$C_1(x,t) = \sum_{i=1}^{n-1} R_i \frac{P_i^2 + Q_i^2}{V_i^2} \text{ [pu]} \quad (4)$$

where  $i$  is any feeder branch;  $n$  is number of network buses;  $R_i$  is the pu resistance of branch  $i$ ;  $P_i$ ,  $Q_i$  are the pu active and reactive powers at the receiving end of branch  $i$ ;  $V_i$  is the pu voltage magnitude at that end.

For the problem of load balance in the system, the following index is defined:

$$C_2(x,t) = \sum_{i=1}^{n-1} R_i \frac{P_i'^2 + Q_i'^2}{S_i^{\max 2}} \text{ [pu]} \quad (5)$$

where  $P_i'$ ,  $Q_i'$  are the pu active and reactive power at the receiving end of branch  $i$  and  $S_i^{\max}$  is the branch pu maximum capacity (apparent power).

The magnitudes of the network node voltages must be within certain pre-defined limits, and also the magnitude of the branch currents must not go beyond the thermal or economic current limit of the respective branches. These constraints, for any radial topological structure  $t_i$ , can be expressed in a compact manner through (3).

## 2.3. Power flow methods

The traditional load flow calculation methods are not adequate to be applied in distribution systems. Thus, it is necessary to adopt a load flow calculation method specifically oriented for radial distribution systems that is fast, that uses low memory resources and that has good convergence features. In [12] there is a discussion and comparison of three load flow calculation methods for radial networks. They are the ladder method, the current summation method and the power summation method. When the load level is increased, the power summation method has better convergence characteristics [13]. Due to this, this work uses that method to determine the value of the objective function and to verify the operational constraints of the network.

*i) Power Summation Method:* This method incorporates two processes in an iteration, one upstream and another one downstream. In the upstream process, a node is taken and the active and reactive power demand from the network (including losses) is determined seen from that node downstream. In other words, an equivalent active and reactive power connected to such node ( $P_i, Q_i$ ) is obtained. This process is made at each node in the network and it is initialized assuming a voltage profile. In the downstream process, it starts with the node that is topologically after the reference bus, with known voltage in magnitude and angle; using the equivalent powers calculated previously, the modules of the voltages in each node downstream are recalculated. Convergence is only checked with the voltage magnitudes. Once these magnitudes have been determined, it is possible to calculate the respective angles. Below, there is an illustration of the algebraic expressions of the method in reference to an  $i$  section of a feeder that goes from a node  $i-1$  to a node  $i$ .

$$V_i^4 + A_i V_i^2 + B_i = 0 \quad (6)$$

$$A_i = 2 (P_i R_i + Q_i X_i) - V_{i-1}^2 \quad (7)$$

$$B_i = (P_i^2 + Q_i^2) (R_i^2 + X_i^2) \quad (8)$$

$$\tan \beta_i = \frac{P_i X_i - Q_i R_i}{P_i R_i + Q_i X_i + V_i^2} \quad (9)$$

$$\beta_i = \text{ang}(V_{i-1}) - \text{ang}(V) \quad (10)$$

$$P_i = P_{Li} + \sum_{N_{Ai}} P_k + \sum_{N_{Ai}} R_k \frac{P_k^2 + Q_k^2}{V_k^2} \quad (11)$$

$$Q_i = Q_{Li} + \sum_{N_{Ai}} Q_k + \sum_{N_{Ai}} X_k \frac{P_k^2 + Q_k^2}{V_k^2} \quad (12)$$

where

$V_{i-1}, V_i$  : Voltage magnitudes at nodes  $i-1$  and  $i$        $P_i, Q_i$  : Equivalent active and reactive power at  $i$   
 $R_i, X_i$  : Resistance and inductive reactance of section  $i$        $P_{Li}, Q_{Li}$  : Active and reactive load at node  $i$   
 $N_{Ai}$  : Set of nodes fed directly from node  $i$        $\beta_i$  : Voltage angle difference between  $i-1$  and  $i$

The iterative method consists on using (11) and (12) in the upstream process and (6), (7) and (8) in the downstream process.

ii) *Simplified Power Summation Method*: It is based in the fact that the active and reactive power losses in any  $i$  section of the network are small compared with the active and reactive power flow through that branch. Expression (6) can be rewritten as follows:

$$V_i^2 + A_i + \frac{P_i^2 + Q_i^2}{V_i^2} (R_i^2 + X_i^2) = 0 \quad (13)$$

The third addend of (13) represents the branch losses and according to what has been presented, it can be disregarded. Thus, the magnitudes of node voltages can be approximated with the following expression:

$$V_i^2 + A_i = 0 \quad (14)$$

The equivalent nodal powers ( $P_i, Q_i$ ) are determined in the same manner as presented, except that in this case losses are not considered, so they will only be estimations. It is possible to observe that this simplified method is non iterative.

Now it is possible to estimate the active power losses and the network's load balance index through (4) and (5), respectively.

### 3. Solution methodology

This work uses an heuristic solution methodology based on a search by branch exchange that allows to drastically reduce the computation time involved in the formulation by Baran and Wu [8], when using the simplified power summation method. This version allows to estimate the electrical condition of the system with a reasonable accuracy. The algorithm allows to determine the switching actions to reduce the network's losses or to balance the load of the system. The final network configuration must be radial and all loads must remain connected.

Figure 1 shows a simple schematic diagram of a primary distribution network. There are switches that are normally closed that allow to supply power to zones located downstream of the respective switches (LL2-LL14) and there are switches that are normally open that allow to connect zones between two feeders (LL5-LL6) and/or laterals that belong to a same feeder. For instance, the network can be reconfigured when the LL6 switch is closed; as this maneuver creates a loop in the network, the LL14 switch must be opened to make the system stay with a radial topological structure. As a result from this maneuver, zone 13007 will be transferred to the 01 feeder. This basic maneuver will be called "branch exchange". In general, it is possible to make more complex maneuver schemes, when applying several successive branch exchanges.

The basic idea of the search scheme when using the "branch exchange" method consists on starting with a feasible tree (parent tree) and successively creating new trees (offspring trees) when doing "one" branch exchange at a time; this operation is always made from the parent tree. Each open switch will originate as many offspring trees as closed switches exist in the respective associated loop; that is to say, the amount of branch exchanges that can be made from the parent tree is equal to the number of closed switches that belong to the loops associated to the open switches. It is necessary to choose the best offspring tree from all the trees; that is to say, the branch exchange with which the highest reduction of the objective function is obtained without infringing the operational constraints of the network. Now, the selected tree is transformed into a parent tree and the process of generating new offspring trees is repeated identically.

The selection process of a new parent tree is repeated until it is not possible to reduce the objective function anymore. To determine the value of the objective function and to verify the operational constraints of the network for a given configuration, a load flow calculation is made through the Power Summation Method. For that purpose both the iterative and the simplified versions were used.

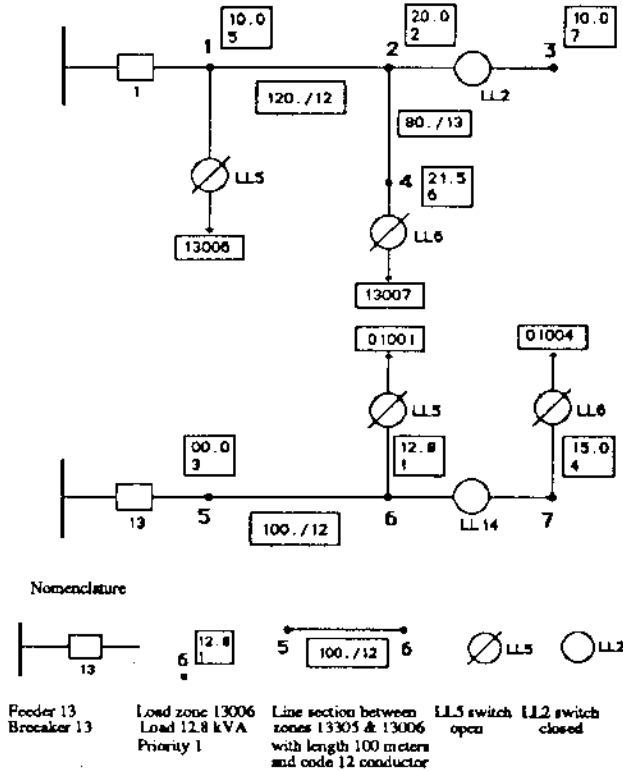


Figure 1. Diagram of a primary distribution network

*i) Some computer considerations:* The solution algorithm proposed was coded in Turbo Pascal 5.5. The radial topological structure of the network is dynamically built using pointer variables. As the number of node offsprings is variable, it was preferred to represent the network tree through a binary tree, that is to say, all the offsprings of a node are stringed through a linear list. As the binary tree is built, the addresses of all pointers that identify each node (records) are saved in a file. This allows to have direct access to any pair of nodes of an element of the tree to make efficient and fast topological changes, preventing the use of a search methodology to locate the nodes; that would be inadequate for practical purposes. An interesting aspect refers to the node addresses; they are not modified with topological changes made. In reported papers, Fortran language has been used, where the topological structure is built by arrays. This makes it difficult to program the simulation of topological changes and the path of the network tree, and ultimately implies a higher computation effort. Nevertheless, the methodology of the binary tree proposed in this work appears as an alternative from the computation standpoint.

#### 4. Application to a test system

The program was evaluated with different test systems. However, due to briefness, here we introduce the results corresponding to a 13.8 kV network with three feeders, forty six nodes, twenty six load points, twenty three line sections, three breakers and twenty seven sectionalizing switches, with eight of them normally open (Figure 2, Appendix).

Two procedures were built, one of them uses the iterative version of the Power Summation Method (exact method) and the other one uses the simplified version (approximate method). Both procedures were tested for different load factors. However, here we only present the results for a unit load factor. Tables I and II show the sequence of maneuvers given by both methods, for the active loss reduction in the network.

From the tables it is possible to see that both methods make the same decisions for the three first maneuvers. The sequence that follows is different. This is explained because initially the loss reductions are large and the errors in the estimations of the approximate method are small enough so the same

maneuvers are chosen for both methods. However, as the loss reduction gets smaller, the estimation of the approximate method is less accurate, originating decisions different from the ones made by the exact method. Another aspect that can be seen in the tables is that both methods give an almost equal total reduction of active losses, even though the maneuver sequence for small power reductions does not coincide.

Computer time requirements are significantly reduced when using the approximate method. An 85% cpu time reduction from 52 sec. to 8 sec. was achieved when minimizing losses for a 70 node system in an IBM/PC AT 16 MHz computer. Similar reductions are also obtained when balancing loads.

Table I. Loss minimization, exact method			Table II. Loss minimization, approximate method		
Sequence of maneuvers			Sequence of maneuvers		
Initial loss: 23.9 (kW)			Initial loss: 23.9 (kW)		
$\Delta P$ (kW)	Close	Open	$\Delta P$ (kW)	Close	Open
15.583	702	289	15.453	702	289
4.992	704	1	4.970	704	1
1.187	703	271	1.183	703	271
0.034	322	256	0.034	322	352
0.003	1	2	0.016	705	266
0.000	256	21	0.075	271	272
			0.002	146	285

## 5. Conclusions

In this work, there is a general formulation of the problem of feeder reconfiguration to reduce active losses and balance the load of a radial electric power primary distribution network under normal operating conditions. Both subproblems are similar and are solved by means of an heuristic solution algorithm based on the method of "branch exchange". The feasibility of a branch exchange is determined through the calculation of the electrical condition of the network; for that purpose, a method of load flow calculation is used, in its iterative and simplified version, called "Power Summation". Once the electrical condition of the network is known for a given branch exchange, it is possible to quantify the objective function and verify the operational constraints of the system.

From the analysis made to test networks, it is possible to conclude that both reconfiguration methods work satisfactorily, revealing the convenience of using the approximate method to perform reconfiguration studies in distribution networks under normal operating conditions, as this method requires a smaller computational effort compared to the exact method, and the solutions delivered by both methods are identical. In general, both methods converge to a local optimum, that is to say, convergence to a global optimum is not guaranteed.

The program developed can be used in systems with or without automation. In non-automated distribution systems, the program can be used as an analysis tool to make planning studies or to make decisions about the modification of the topological structure of the network, for example during different seasons of the year.

## 6. Acknowledgement

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APPENDIX

Table III.- Loads in the test system (power factor 0.8, load factor 1.0)

Code	Zone	kVA
1	17520	45.0
2	17524	75.0
3	17530	232.5
4	17532	600.0
5	17560	1500.0
6	17561	300.0
7	17570	750.0
8	19319	30.0
9	19340	112.5
10	19370	105.0
11	19373	45.0
12	19383	600.0
13	19390	700.0
14	19393	450.0
15	19394	412.5
16	21141	75.0
17	21142	75.0
18	21148	112.5
19	21149	75.0
20	21153	75.0
21	21160	90.0
22	21165	75.0
23	21170	400.0
24	21172	120.0
25	21174	112.5
26	21176	75.0

Table IV.- Conductors of the test system

Code	Material	Size	R ( $\Omega/km$ )	X ( $\Omega/km$ )	$I_{max}$ (A)
1	Cu	8 AWG	2.3617	0.5239	90
2	Cu	7 AWG	1.8707	0.5152	110
3	Cu	6 AWG	1.4854	0.5065	120
4	Cu	5 AWG	1.1777	0.4878	140
5	Cu	4 AWG	0.9341	0.4890	170
6	Cu	3 AWG	0.7408	0.4804	190
7	Cu	2AAWG	0.5935	0.4654	240
8	Cu	1 AWG	0.4705	0.4567	270
9	Cu	1/0	0.3772	0.4499	310
10	Cu	2/0	0.2989	0.4412	360
11	Cu	3/0	0.2374	0.4325	420
12	Cu	4/0	0.1883	0.4232	480
13	Cu	250 MCM	0.1597	0.4095	540
14	Cu	300 MCM	0.1336	0.4027	610
15	CAA	6 AWG	2.4736	0.5288	100
16	CAA	5 AWG	1.9764	0.5239	120
17	CAA	4 AWG	1.5973	0.5201	140
18	CAA	3 AWG	1.2865	0.5214	160
19	CAA	2 AWG	1.0503	0.5239	180
20	CAA	1 AWG	0.8577	0.5239	200
21	CAA	1/0	0.6961	0.5183	230
22	CAA	2/0	0.5552	0.5089	270
23	CAA	3/0	0.4493	0.4965	300
24	CAA	4/0	0.3679	0.4717	340
25	CAA	266.8 MCM	0.2393	0.3996	460
26	CAA	336.4 MCM	0.1902	0.3909	530
27	CAA	397.5 MCM	0.1609	0.3847	590
28	CAA	477 MCM	0.1342	0.3778	670
29	CA	6 AWG	2.4301	0.5022	100
30	CA	4 AWG	1.5289	0.4959	134
31	CA	3 AWG	1.2119	0.4759	155
32	CA	2 AWG	0.9633	0.4672	180
33	CA	1 AWG	0.7645	0.4585	209
34	CA	1/0	0.6047	0.4483	242
35	CA	2/0	0.4792	0.4410	282
36	CA	3/0	0.3810	0.4323	327
37	CA	4/0	0.3021	0.4236	380
38	CA	266.8 MCM	0.2399	0.4148	441
39	CA	336.4 MCM	0.1908	0.4019	514
40	CA	397.5 MCM	0.1610	0.3956	575
41	CA	477 MCM	0.1342	0.3888	646

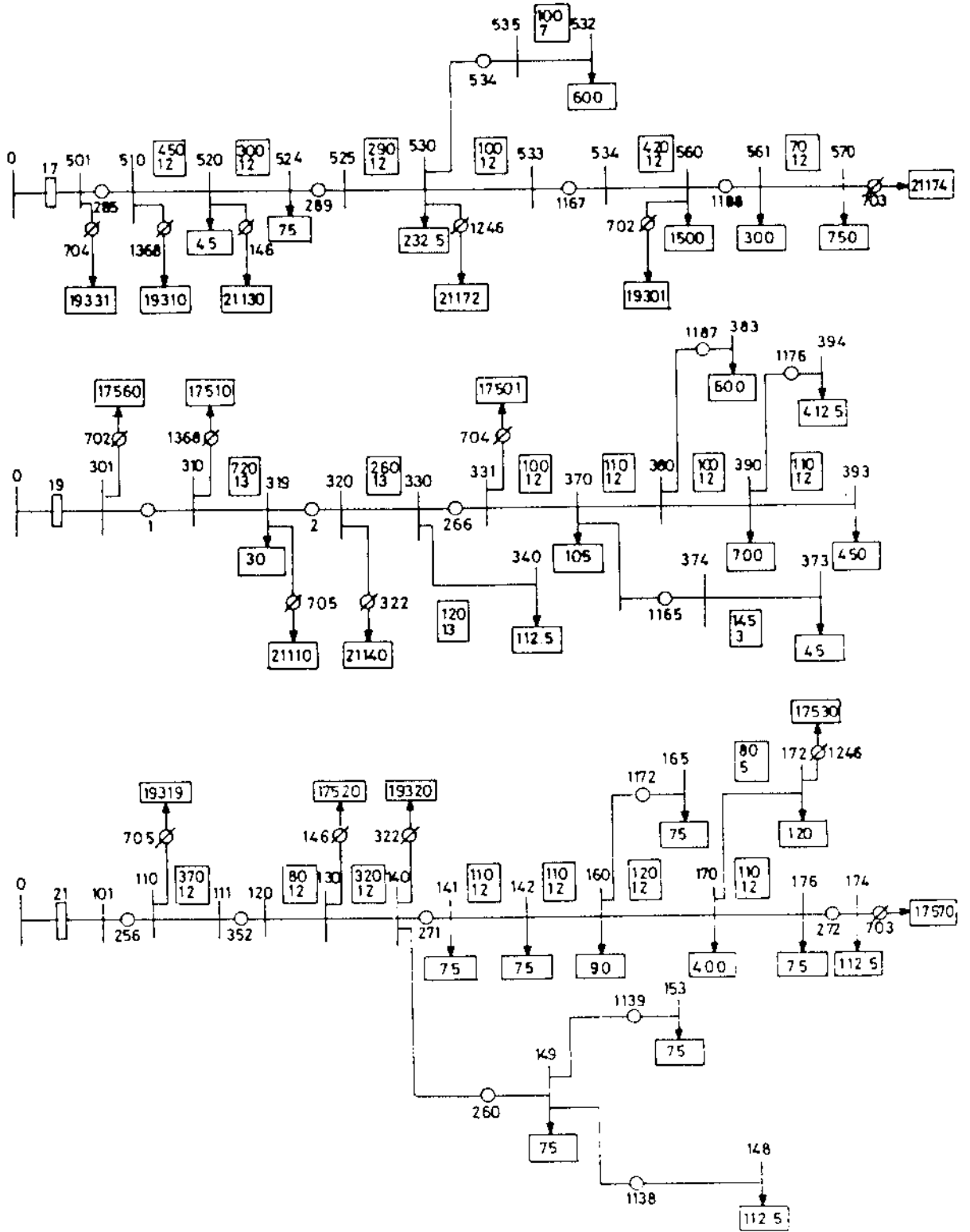


Fig. 2.- Test system