

STABILITY ANALYSIS AND PERFORMANCE CHARACTERISTICS OF AN OPEN-LOOP PWM VAR COMPENSATOR

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Abstract

An open-loop, three-phase, transistorized var compensator, with a single PWM pattern is analyzed. The var generation is controlled by shifting the fixed PWM pattern with respect to the ac-mains voltage. The system presents natural stability and then there is no need to control the capacitor dc voltage, which takes a particular stable voltage for each power factor operation condition. This particular behaviour is studied and a complete stability analysis, based on the d-q frame is developed. From this analysis, the circle diagram of the var compensator is obtained, which gives all the operating characteristics related to the dc voltage, power angle operation and amount of reactive power generated. The converter presents a very large capacity to generate reactive power. The main characteristics of this compensator are its simplicity and its strong stability. Experimental results confirm the operation characteristics given by computer simulations and analyses.

Introduction

The development of powerful solid-state devices has allowed the implementation of many kinds of converters, such as power rectifiers, power inverters and others [1]. The same electronic components have also been used for reactive power compensation. In this field of application, the first step was done by using thyristor power converters and the line commutated principle [2,3]. However, the inherent mode of operation of this kind of converter produces the generation of unwanted harmonics. This problem has been reduced by using force-commutation [4,5].

The advantages of using force-commutated var compensators and special PWM techniques have already been confirmed. Its practical implementation is today possible in the field of medium and high levels of reactive power compensation thanks to the development of GTO's, high power bipolar transistors and other electronic devices. These PWM techniques permit a considerable reduction in harmonic distortion. Besides, the generation of reactive power does not depend on the size of the capacitors used [6].

However, force commutated PWM devices are much more complicated than thyristor var compensators, because they need special patterns of modulation, control blocks and feedback systems. Searching for a simple design, this paper analyzes the behaviour of a voltage source type, PWM var compensator, which works without any feedback voltage or current and also with a fixed PWM pattern. Some previous works have introduced the idea of using a fixed PWM pattern [7], but there is no mention about the inherent capability of maintaining stable points of operation in open-loop conditions, which is not possible unless the equivalent circuit used considers the internal power losses [8]. Practical devices always have internal losses and then, as it will be demonstrated here, a stable operation condition in open-loop does exist. The analytical demonstrations are confirmed here by using digital computer simulations and experiments with a 2-Kvar PWM compensator.

Operation Principle

The heart of the power var compensator is a conventional three-phase modulator with gate turn-off switches, as shown in figure 1. The switches of the modulator are controlled by a fixed PWM pattern which is synchronized with the mains. The reactive power is controlled by changing the phase angle θ between the mains and the PWM pattern. Each time the θ angle (reference) is changed the dc capacitor voltage changes, keeping a new stable operation voltage. Because the dc voltage changes, the amplitude of the fixed PWM does too, altering the fundamental ac voltage V_{mod}

at the converter terminals, which is produced by the action of the power switches. The amount of reactive power generated (or absorbed) is basically dependent on the difference of amplitude between the mains voltage V and V_{mod} . The internal losses of the modulator plays an important role in keeping the dc capacitor voltage in a stable value. If one assumes for a moment that the compensator is lossless, the capacitor dc voltage could increase until its complete collapse. The next step is to prove that the open-loop var compensator is always stable under real conditions.

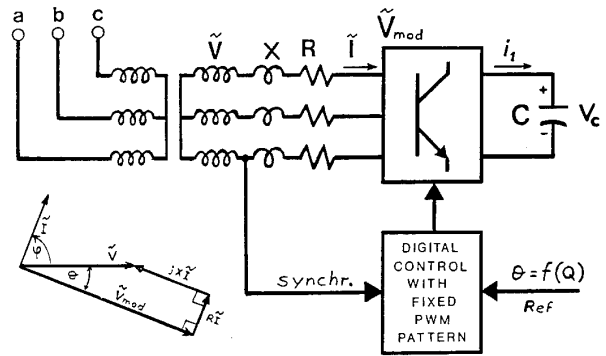


Fig.1 : The open-loop PWM var compensator

Stability Analysis

One simple way to analyze the stability of the open-loop var compensator is through the d-q frame. The figure 2 shows the phasor diagram of the modulator in the d-q frame.

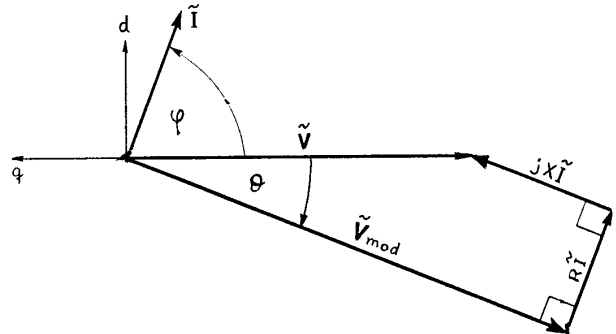


Fig. 2 : the phasor diagram of the var compensator

Each element of figure 2 represents:

- V = mains phase-to-neutral voltage
- V_{mod} = fundamental voltage of the PWM pattern
- $X=wL$ = input reactance of the compensator
- R = input resistance
- I = ac var compensator current
- d-q = reference frame.

The losses of the compensator depends on the switching frequency (which is constant), on the dc voltage and fundamentally

on the ac currents. With an acceptable approximation, they can be represented by the input resistance R , through the term $3RI^2$. Because the PWM pattern is fixed, the relation between the dc voltage v_c and the instantaneous value of the amplitude of V_{mod} is constant:

$$|v_{mod}| = K_v \cdot v_c \quad (1)$$

The dynamic equation of the modulator can be written in d - q axis as:

$$\begin{bmatrix} L & 0 \\ 0 & L \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} R & -X \\ X & R \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} V_{mod_d} \\ V_{mod_q} \end{bmatrix} \quad (2)$$

where

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 0 \\ -\sqrt{3} V \end{bmatrix}$$

represents the mains voltage in the d - q frame. Introducing eq. (3) into (2) and because $x = wL$, one gets:

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = -\frac{1}{L} \begin{bmatrix} V_{mod_d} \\ V_{mod_q} + \sqrt{3} V \end{bmatrix} - \begin{bmatrix} \frac{B}{L} & -w \\ w & \frac{B}{L} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (4)$$

V_{mod_d} and V_{mod_q} are related through the θ angle, which is the reference:

$$\frac{V_{mod_q}}{V_{mod_d}} = -\text{ctg } \theta = \sigma_v \quad (5)$$

The instantaneous value of the amplitude of V_{mod} , in terms of its d - q components is expressed as:

$$|V_{mod}| = \frac{1}{\sqrt{3}} \sqrt{V_{mod_d}^2 + V_{mod_q}^2} \quad (6)$$

The combination of eqs. (1), (5) and (6) permits one to find expressions which relate V_{mod_d} and V_{mod_q} with the dc voltage v_c :

$$V_{mod_d} = \frac{\sqrt{3} k_v}{\sqrt{1 + \sigma_v^2}} v_c = K_d \cdot v_c \quad (7)$$

$$V_{mod_q} = K_d \sigma_v v_c = K_q \cdot v_c$$

By substituting eqs. (7) and (8) into (4) yields:

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \end{bmatrix} = -\frac{1}{L} \begin{bmatrix} K_d v_c \\ K_q v_c + \sqrt{3} V \end{bmatrix} - \begin{bmatrix} \frac{B}{L} & -w \\ w & \frac{B}{L} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (9)$$

There are two equations in (9) and three unknown values: v_c , i_d and i_q . A third equation is required. The power balance equation and the dc capacitor voltage equation solve the problem:

$$v_c i_1 = V_{mod_d} \cdot i_d + V_{mod_q} \cdot i_q \quad (10)$$

$$i_1 = C \frac{dv_c}{dt} \quad (11)$$

By substituting (7), (8) and (11) into eq. (10) results:

$$C \frac{dv_c}{dt} = K_d i_d + K_q i_q \quad (12)$$

Equation (12) is the third equation required to solve the problem. Now we have three equations for three unknown values: v_c , i_d and i_q :

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ v_c \end{bmatrix} = \begin{bmatrix} \frac{B}{L} & w & -\frac{K_d}{L} \\ -w & -\frac{B}{L} & -\frac{K_q}{L} \\ \frac{K_d}{C} & \frac{K_q}{C} & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ v_c \end{bmatrix} - \begin{bmatrix} 0 \\ \sqrt{3} V \\ 0 \end{bmatrix} \quad (13)$$

$$\dot{X} = [A] \cdot X + B \quad (14)$$

where

$$[A] = \begin{bmatrix} \frac{B}{L} & w & -\frac{K_d}{L} \\ -w & -\frac{B}{L} & -\frac{K_q}{L} \\ \frac{K_d}{C} & \frac{K_q}{C} & 0 \end{bmatrix} \quad (15)$$

Stability. The stability of the system can be found through the eigenvalues of the $[A]$ - matrix:

$$\det \{S[I] - [A]\} = 0 \quad (16)$$

or:

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0 \quad (17)$$

Applying the Routh's Criterion ($a_1 > 0$, $a_2 > 0$, $a_3 > 0$) one finds that the system is stable if and only if:

$$R > 0 \quad (18)$$

$$L > 0 \quad (19)$$

(8) The consequence of this result is that, with respect to R , the open-loop var compensator is always stable, because it is not possible to have a practical device with theoretically zero losses ($R = 0$). The inductance L is always needed in this kind of converter. Another important consequence of this analysis is that the stability is independent of the size of the dc capacitor. The capacitor has to be dimensioned based on other design restrictions such as amount of ripple at the dc link.

Steady - state Operation

Under steady-state operation $\dot{X} = 0$ and then:

$$K_d v_c + R i_d - X i_q = 0 \quad (20)$$

$$K_q v_c + X i_d + R i_q = -\sqrt{3} V \quad (21)$$

$$K_d i_d + K_q i_q = 0 \quad (22)$$

From eqs. (20), (21) and (22) one gets an expression for V_C under steady-state:

$$V_C = \frac{-\sqrt{3} V (R K_q + X K_d)}{R(K_d^2 + K_q^2)} \quad (23)$$

but because of eqs. (7), (8) and (5)

$$V_C = \frac{V (R \cdot \text{ctg } \theta - X) \text{sen } \theta}{R K_V} \quad (24)$$

or

$$V_C = \frac{V (R \cos \theta - X \text{sen } \theta)}{R K_V} \quad (25)$$

K_V is a constant which depends on the switching pattern of the compensator. By substituting (1) into (24) it yields:

$$|V_{\text{mod}}| = \frac{V(\cos \theta - \frac{X}{R} \text{sen } \theta)}{R} \quad (26)$$

The circle diagram

Equation (26) represents the equation of a circle in polar coordinates. This circle diagram shown in Figure 3, relates the magnitude of V_{mod} (and hence the magnitude of V_C) for different values of the reference θ . The direction of I is in the direction of RI in the diagram.

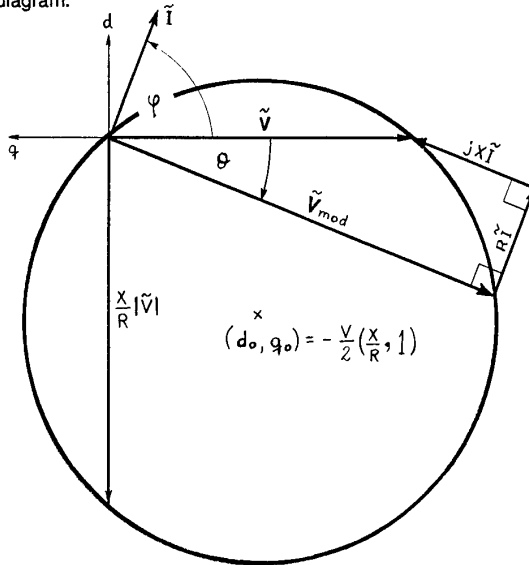


Fig. 3 : The circle diagram of the var compensator

It can be observed from the circle diagram of the compensator that the bigger is the X/R rate, the bigger is the diameter of the circle. In such a case, the system is very sensitive because small changes in the θ reference produce big variations in $|V_{\text{mod}}|$ and consequently in V_C . When V_{mod} leads the mains V , the magnitude of V_{mod} is smaller than the magnitude of V and the compensator absorbs reactive power (θ positive). When θ is negative, then V_{mod} lags the voltage V and then the system generates reactive power because $|V_{\text{mod}}|$ becomes bigger than $|V|$. The amount of reactive power the converter can generate depends on the magnitude of I_d which is in quadrature with the mains voltage.

The circle diagram also permits one to realize what would happen in the event that R were neglected in the analysis. In such a situation we would have a "circle diagram with an infinite radius, and then no possible equilibrium point for V_{mod} could be found. As a consequence V_C neither would have a stable voltage and the system would collapse.

Reactive power generation. It depends on I_d magnitude. When $I_d \neq 0$ then the compensator generates reactive power. From eqs. (20), (21), and (22) one gets:

$$I_d = \frac{\sqrt{3} V}{R \left(\frac{K_q}{K_d} + \frac{K_d}{K_q} \right)} \quad (27)$$

But because of (5), (7) and (8):

$$\therefore \frac{K_q}{K_d} = -\text{ctg } \theta \quad (28)$$

$$I_d = -\frac{\sqrt{3} V \text{sen } 2\theta}{2R} \quad (29)$$

In the d - q frame the reactive power is:

$$Q = -V_q \cdot I_d = \sqrt{3} V \cdot I_d \quad (30)$$

and finally from (29) and (30):

$$Q = -\frac{3V^2}{2R} \text{sen } 2\theta \quad (31)$$

Plotting the magnitude of Q as a function of the reference angle θ one gets the graphic shown in Figure 4.

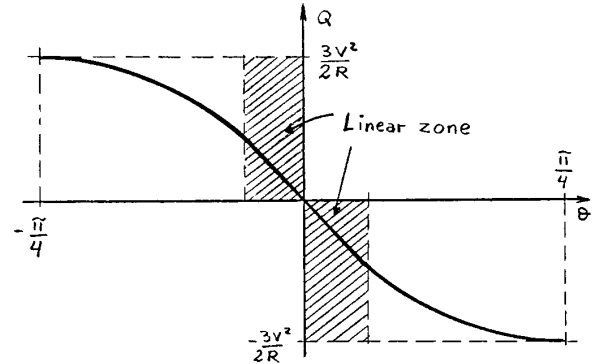


Fig. 4 : Reactive Power vs θ angle characteristic

It can be observed from Figure 4 that the maximum reactive power that can be produced is equal to

$$Q_{\text{MAX}} = \frac{3}{2} \frac{V^2}{R} \quad (32)$$

And it occurs when $\theta = -45^\circ$. However, the operation limits of the var compensator should be confined to the "linear zone" defined in Figure 4. In this zone, the amount of reactive power generated (or absorbed), is almost proportional to the θ angle.

Power losses. The power losses of the converter are unavoidable but at the same time necessary for the stability in open-loop operation. They permit a very simple implementation and control of the reactive power through an almost linear variation in the reference (θ angle). The power losses can also be evaluated in the d - q frame, and they depend on the magnitude of I_q .

$$P_L = V_q \cdot I_q = -\sqrt{3} V I_q \quad (33)$$

I_q can be obtained from (20), (21) and (22), and with the help of (5), (7) and (8):

$$I_q = -\frac{\sqrt{3} V}{R} \sin^2 \theta \quad (34)$$

Combining (33) with (34)

$$P_L = \frac{3 V^2}{R} \sin^2 \theta \quad (35)$$

Because of the quadratic characteristic of eq. (35), power losses are always positive and they have a maximum value at $\theta = \pm 90^\circ$. Nevertheless, the normal operation angle should be small. Figure 5 shows the power losses vs θ angle characteristic.

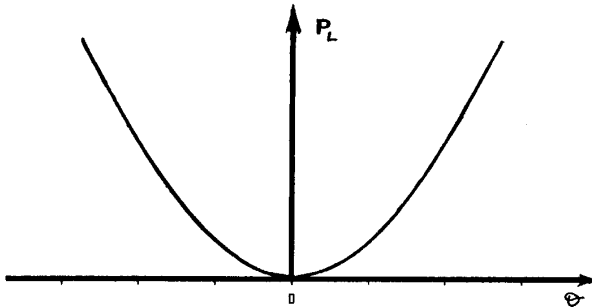


Fig. 5 : Power losses vs θ angle characteristic

Because of the term $\sin^2 \theta$, power losses becomes important when the θ angle is bigger. For small θ angles, the relation between Q and P_L becomes better. From eqs. (31) and (35):

$$\left| \frac{Q}{P_L} \right| = \frac{\sin 2\theta}{2\sin^2 \theta} = |\operatorname{ctg} \theta| = |\sigma_v| \quad (36)$$

Plotting eq. (36) one gets the relation between the generation of reactive power and the power losses. It is shown in Figure 6.

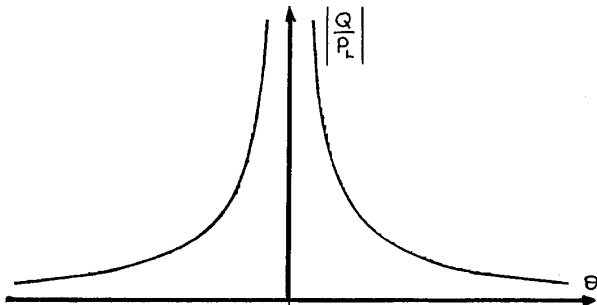


Fig. 6 : $|Q/P_L|$ vs θ angle

From Figure 6 it can be concluded that small θ angles of operation bring better results.

Power losses at the dc link. Until now, the power losses of the dc capacitor have been neglected. Nevertheless, they can be considered into the analysis adding a resistor in parallel with the dc capacitor. In such a case, the capacitor equation (eq. (11)) becomes:

$$C \frac{dv_c}{dt} = i_1 - \frac{v_c}{R_c} \quad (37)$$

Repeating all the previous procedures, the matrix $[A]$ (eq. (15)) becomes:

$$[A] = \begin{bmatrix} -\frac{B}{L} & w & -\frac{K_d}{L} \\ -w & -\frac{B}{L} & -\frac{K_q}{L} \\ \frac{K_d}{L} & \frac{K_q}{L} & -\frac{1}{R_c C} \end{bmatrix} \quad (38)$$

Similar analysis of stability permits one to realize that the system is also stable when $R > 0$.

Simulations and experimental results

The performance characteristics of the open-loop var compensator have been verified by digital computer simulations. The valve - by - valve digital simulation program developed for the compensator, is based on treating the three-phase bridge as a piece-wise circuit problem. The ON-OFF states of each of the six power-switches give rise to eight possible circuit topologies. The instants of switchings of the semiconductors are defined by the switching pattern adopted, which is unique. In this form, a time sequence of eight circuit topologies is generated. The switching pattern is synchronized with the mains, forming a fixed θ angle. The differential equations arising from the Kirchhoff Voltage and Current Laws for the topologies are solved by the requirements that the flux linkages and electric charges must be continuous. Changing the θ angle, different conditions of operation can be obtained.

The advantages of working with simulations are that they allow to visualize the behaviour of very large var compensators and also they permit a good tuning in the design.

For experimental results, a 2 - kvar compensator has been implemented. It was built using modular darlington bipolar transistors and simplified driver circuits using hybrid components. The control block is basically composed by a simple EPROM with only one pattern stored, which is commanded by the frequency mains. The θ angle reference is controlled by a digital adder and is modified manually. Input inductances have been constructed using toroidal-core ferrites with a small airgap to avoid saturation. The dc side has a 1.5 mF electrolytic capacitor. This value has been adopted only for dc ripple considerations. The start-up of the system is simple: when connected, the power transistors remain inhibited and the capacitor voltage is charged through the antiparallel diodes of the transistors. When the dc capacitor is fully charged, the control system, which is already producing the synchronized switching pattern, is automatically connected to the transistor drivers. At this moment the compensator begins its operation.

Lagging operation. The oscillograms of figure 7 show a computer simulation and an experimental result of a lagging operation. The magnitude of the current is 6 A and the mains voltage V is 90 V. The switching pattern in both cases is exactly the same: an over-modulated sinusoidal PWM with triangular carrier. The reactive power absorption depends basically on the current ratings of the power semiconductors.

Leading operation. The oscillograms of Figure 8 show a computer simulation and an experimental result of a leading operation. The magnitude of the current is 6,5 A and the mains voltage is 100 V. The switching pattern is the same in both cases and also the same as the one used in previous lagging power factor oscillograms. During leading power factor operation, the compensator works with a modulated voltage V_{mod} , bigger than the mains voltage, V . The dc voltage V_c , depends directly on V_{mod} and hence it is also bigger than the values it take under lagging power factor operation. The bigger is the amount of Q generated, the bigger is this dc voltage. As a consequence, the generation of big amounts of reactive power

would produce voltage stresses in the power switches and also in the dc capacitor. For this reason, the reactive power generation, depends on both, the current and voltage ratings of the electronic switches, and also on the voltage rating of the dc capacitor.

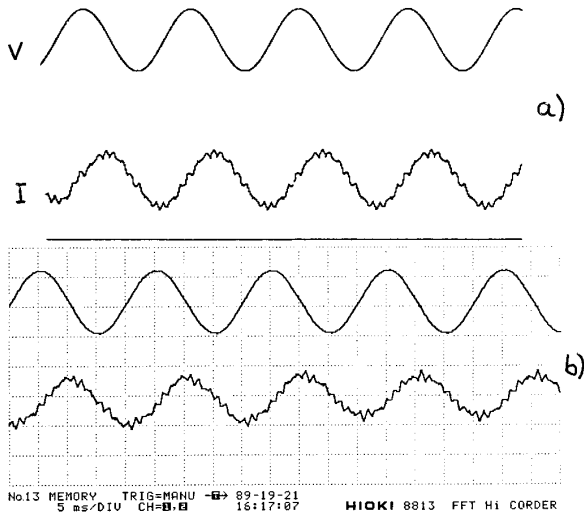


Fig. 7: Lagging oscillograms
a) Simulation
b) Experiment

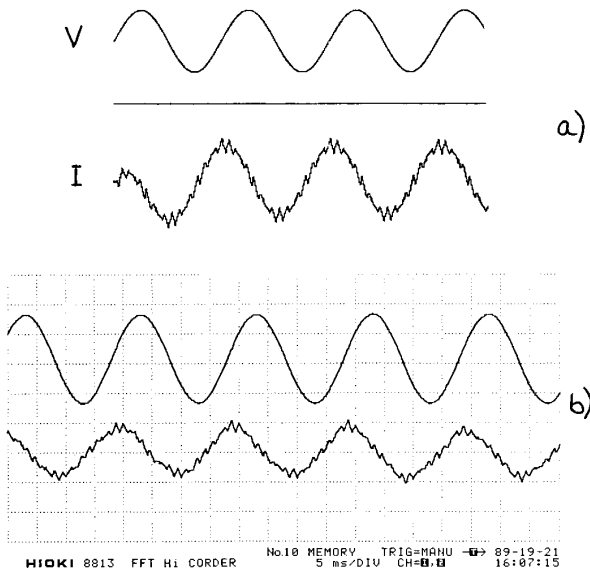


Fig. 8: Leading oscillograms
a) Simulation
b) Experiment

Transient Response. It has been observed that the dynamic behaviour of the open-loop var compensator is fundamentally dependent on the size of the dc capacitor. The smaller the dc capacitor size, the faster the transient response. The simulations of Figure 9 show the behaviour of the compensator under two different situations: the first, with a 1.5 mF and the second with a 0.5 mF dc capacitor. In the second case, the dc voltage ripple becomes visible, but in such a case it is a matter of improving the switching pattern to keep the ripple low with small dc capacitors. Faster response and low ripple content have to be balanced properly.

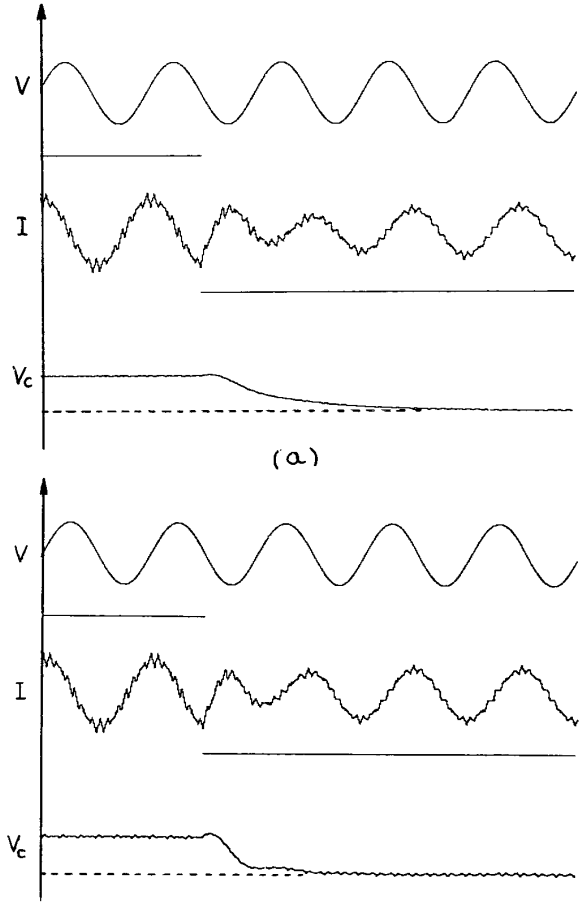


Fig. 9 : Step reversal simulation from leading to lagging current
a) $C = 1.5 \text{ mF}$
b) $C = 0.5 \text{ mF}$

The Figure 10 shows an experimental result using a 1.5 mF dc capacitor. In this experiment, the ac current of the laboratory prototype changes from 8 A leading to 9 A lagging. As it can be seen, the response is not as good as in the simulations. This is mainly because of the poor regulation of the mains used in the laboratory. Other problem is the synchronization signal which have to be improved.

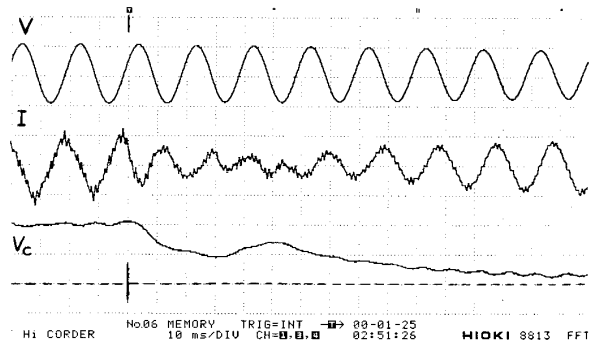


Fig. 10 : Step reversal experiment from leading to lagging current

Conclusions

The stability and performance characteristics of an open-loop, voltage source, PWM var compensator has been analyzed. The circle diagram of the compensator has been developed, which permit to realize that the dc voltage V_c always find a stable value of operation if one consider the natural power losses of practical devices. The sensitivity of the system depends mainly on the X/R ratio, and the dynamic response depends on the size of the dc capacitor. Simulations and experiments confirm the results obtained in the theoretical analysis.

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References

- [1] T. Salzman and A. Weschta, "Progress in Voltage Source Inverters and Current Source Inverters with Modern Semiconductor Devices", IEEE Ind. Appl. Soc. Annual Meeting, Oct. 1987, pp 577-583.
- [2] D.E. Steeper and R.P. Stratford, "Reactive Compensation and Harmonic Suppression for Industrial Power Systems, Using Thyristor Converters", IEEE Trans. on Ind. Appl., Vol. IA-12 N°3, May/Jun 1976.
- [3] P.A. Caldeira and E.H. Watanabe, "Compensation of Power Factor Rectifier Systems Utilized in Oil Drilling Rigs", IEEE Trans on Ind. Appl., Vol. 24 N°2, March/April 1988, pp 301-307.
- [4] B.T. Ooi, J.C. Salmon, J.W. Dixon and A.B. Kulkarni, "A three-phase Controlled Current PWM Converter with Leading Power Factor", IEEE Trans. on Ind. Appl. Vol. IA-23 N°1, Jan/Feb 1987, pp 78-84.
- [5] Y. Sumi, T. Hasegawa, M. Yano, K. Ikeda and T. Matsura, "New Static var Control Using force-commutated inverters", IEEE Trans. Power Appl. Syst., Vol. PAS-100, pp 4216-4223, Sept. 1981.
- [6] N. Mohan, T.M. Undeland and W.P. Robbins, Power Electronics. New York: WILEY, 1989, ch 16, pp 401-402.
- [7] L.T. Morán, P.D. Ziogas and G. Joos, "Analysis and Design of a Three-Phase Synchronous Solid-State Var Compensator", IEEE Trans. on Ind. Appl. Vol. 25 N°4, July/Aug 1989, pp 598, 608.
- [8] J.W. Dixon and B.T. Ooi, "Series and Parallel Operation of Hysteresis Current Controlled PWM Rectifiers", IEEE Trans. on Ind. Appl. Vol. 25 N°4, July/Aug 1989, pp 644-651.