SELECTING THE BEST POINT OF CONNECTION FOR SHUNT ACTIVE FILTERS IN MULTI-BUS POWER DISTRIBUTION SYSTEMS

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ABSTRACT. Since shunt active power filters operate as controlled current sources injecting current harmonic components to the power distribution system, the point of connection must be carefully selected so the generated harmonic components flow to the nonlinear loads and do not propagate through the distribution system. In this paper an analytical procedure based in the power distribution system voltage and current transfer matrices is derived providing a user friendly and general tool to determine the most adequate point of connection for shunt active power filters, improving the compensation effectiveness. The validity of the developed method is verified by theory and computer simulation. Application to a real multi-bus power distribution system is presented.

I.- INTRODUCTION

Active power filters have become an interesting and effective solution for dynamic reactive power and current harmonic compensation in power distribution systems [1]. With the development of Flexible AC Transmission Systems (FACTS) and Flexible, Reliable, and Intelligent Electrical Energy Delivery System (FRIENDS) concepts in transmission and distribution [2], active filters will play an important role in the compensation, performance, and the power quality associated with such systems. So far, active power filters have been analyzed in terms of principles of operation, control requirements, and compensation characteristics [3] – [6]. Compensation performance in real power distribution systems has been discussed in terms of the results obtained in voltage compensation, power factor correction and current filtering for a specific load [7]- [8]. Few analysis can be found related with the active power filter compensation performance in large power distribution systems [9] – [10].

The main objective of shunt active power filters is the elimination of current components that affect power distribution efficiency, such as harmonics and reactive components. If these current components are supplied by active power filters connected in a strategic bus, it is possible to confine the circulation of these unwanted current components in a specific region of the power distribution system, therefore improving the overall system efficiency and reliability. The selection of the active filter point of connection in multi-bus power distribution systems is not trivial, and can affect current and voltage compensation performance significantly.

In Akagi paper [7] it was shown that compensation performance of shunt active power filters depends on the load characteristics, and demonstrated that shunt active power filters are more suitable for the compensation of current source type of loads. The compensation performance for voltage source type of loads is not fully satisfied with shunt compensation.

This type of analysis [7] can be easily done in small power distribution systems, or in equivalent circuit represented by the voltage source, Thévènin equivalent impedance and the nonlinear load. However, in real power distribution systems composed of a large number of buses and different types of loads, the compensation performance and effectiveness of a shunt active power filter strongly depends on the point of connection. The selection of this point is not obvious, and must be done carefully by considering the power system topology and load distribution. It is important to note, that if the shunt active power filter is not located properly it will contribute to increase the current and voltage distortion by injecting harmonics that will circulate in all the distribution system, increasing the probability of resonance generation.

In this paper, a procedure based on the power distribution current and voltage transfer matrices is developed in order to exactly determine the more effective point of connection of a shunt active power filter. The proposed method is based on circuit equations and applied to multi-bus power distribution systems. The basic advantage of the proposed method is that can be applied to any power distribution system configuration, since it uses standard mathematical algorithms normally employed in power system analysis. Simulated results prove the advantages of the method and its effectiveness when it is applied to a power distribution system composed of a large number of buses.

II.- BASIC EQUATIONS

A power distribution system can be represented by a set of algebraic equations in the frequency domain. Each equation represents the relation between a number of dependent

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variables $\boldsymbol{X(s)}$ with equal number of current sources $\boldsymbol{I(s)}$, and also with voltage sources $\boldsymbol{V(s)}$ through two power distribution system transfer matrices $\boldsymbol{H_{nodeI}}$ and $\boldsymbol{H_{nodeV}}$ respectively.

The bus currents I_{bus}, are equal to the Norton equivalent current (I_{Norton}) plus the currents generated by the nonlinear loads (I_{NL}) and the currents supplied by controlled current sources, such as the one injected by shunt active power filters. Each of these currents are affected by their respective incident matrix. It is well known that

$$Y_{bus} * V_{bus} = I_{bus} \qquad (1)$$

This equation can be represented in terms of the different current components multiplied by the respective incident matrix, A, that is

$$Y_{bus} * V_{bus} = A_{INL}^{t} * I_{NL} + A_{INorton}^{t} * I_{Norton} \qquad (2)$$

where A_{INL} and $A_{INorton}$ are the incident matrices of the different current components of the bus current. According to the property of the branch incident matrix:

$$A_{branch} = - A_{INorton} \quad \text{and} \quad I_{Norton} = Y_p * V_P \qquad (3)$$

Where Y_p is the primitive admittance matrix and V_p is the primitive voltage vector. By replacing (3) in (2) the following equation is obtained:

$$Y_{bus} * V_{bus} = A_{INL}^{t} * I_{NL} + (-A_{branch}^{t}) * (Y_p * V_p) \qquad (4)$$

By pre-multiplying (4) by Y_{bus}^{-1}

$$V_{bus} = Y_{bus}^{-1} * A_{INL}^{t} * I_{NL} + Y_{bus}^{-1} * (-A_{branch}^{t}) * Y_p * V_p \; (5)$$

The system distribution transfer matrices of currents and voltages are defined by:

$$H_{nodeI} = Y_{bus}^{-1} * A_{INL}^{t} \qquad (6)$$

$$H_{nodeV} = Y_{bus}^{-1} * (-A_{branch}^{t}) * Y_p \qquad (7)$$

Finally, by replacing (6) and (7) in (5), the equation for the system bus voltages is obtained.

$$V_{bus} = H_{nodeI} * I_{NL} + H_{nodeV} * V_p \qquad (8)$$

By definition the branch voltage is related to the bus voltage by the branch incident matrix (A_{branch}), therefore:

$$V_{branch} = A_{branch} * V_{bus} \qquad (9)$$

By replacing (8) in (9), then:

$$V_{branch} = A_{branch} * H_{nodeI} * I_{NL} + A_{branch} * H_{nodeV} * V_p \; (10)$$

The branch current, I_{branch}, is equal to the product between the primitive admittance (Y_p) and the respective branch voltage (V_{branch}). If the branch does not have a voltage source connected, the respective primitive voltage V_p is zero, but if the voltage source exists, then V_p must be added to the branch voltage, therefore:

$$\begin{aligned} I_{branch} = {} & Y_p * A_{branch} * H_{nodeI} * I_{NL} \\ & + [\, Y_p * A_{branch} * H_{nodeV} + Y_p \,] * V_p \qquad (11) \end{aligned}$$

The following branch matrices are defined:

$$H_{branchI} = Y_p * A_{branch} * H_{nodeI} \qquad (12)$$

$$H_{branchV} = Y_p * A_{branch} * H_{nodeV} + Y_p \qquad (13)$$

Finally the branch current can be obtained from:

$$I_{branch} = H_{branchI} * I_{NL} + H_{branchV} * V_p \qquad (14)$$

The current matrices H_{nodeI} and $H_{branchI}$ relate the branch voltage and branch current that are induced when a current is injected in a different bus of the power distribution system. This characteristic is very useful for the analysis of active compensation effectiveness in a multi-bus power distribution system, since by solving (14), the most effective active power filter point of connection can be found, and also, for this specific point of connection, the effects in current and voltage distortion in all the other buses of the power distribution system can be obtained.

The method is valid for passive and active filters analysis. The best point of connection for passive filters will be defined avoiding resonances created with inductive elements connected in the power system. However, in the case of active power filtering, the compensation effectiveness can change, since the shunt active power filter operate as current controlled sources, therefore generating current harmonics. This can increase harmonic distortion in power systems, if the current harmonics do not circulate to the non-linear load, in case the point of connection is not selected properly.

III.- DERIVING THE OPERATING CHARACTERISTICS OF A POWER DISTRIBUTION SYSTEM.

The solution of the branch transfer matrices shown in (12) and (13) allows evaluation of the principal operating characteristics of a power distribution system. In fact, the analysis of the system resonances can be done by computing H_{nodeI} with all the current sources connected to the power distribution system, and by plotting each matrix column element for different frequency values. This procedure finds the system resonant frequencies and the buses that are more affected. The system current transfer matrix H_{nodeI} helps to determine the voltage sensitivity of a branch. This important characteristic can be used to find the bus that increases active compensation effectiveness in a power distribution system. This analysis can be done by deriving H_{nodeI} for each bus in which the active power filter can be connected. Once H_{nodeI} is obtained the value of each row element for different frequency values must be calculated. The analysis of the matrix row elements as a function of the frequency allows finding the buses in which the voltage is more affected by the active power filter current injection. This result helps to determine the active power filter best point of connection.

The procedure described in Section II is explained in more detail with the following example. A small power distribution system composed of 3 buses is shown in Fig.1.-

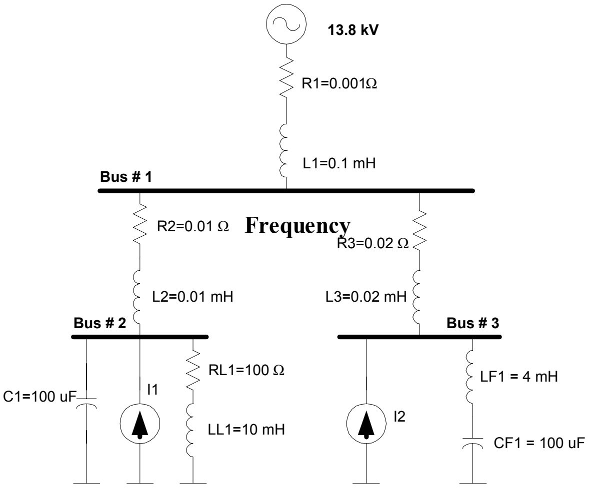


Fig. 1. Sample system one line diagram for voltage sensitivity analysis.

The two current sources I_1 and I_2 represent the non linear loads. The primitive admittance matrix, Y_p, of the power distribution system shown in fig. 1 is equal to:

$$\begin{bmatrix} 0.0008-j0.9095 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0083-j0.9094 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0041-j0.4547 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0045-j0.005 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0+j0.996 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0-j0.0232 \end{bmatrix}$$

where the matrix elements are expressed in per unit with respect to 13.8 kV and 10 MVA base values, and 50 Hz. The system branch incident matrix is equal to:

$$A_{branch} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The dimension of the branch incident matrix is $r \ x \ n$, where r is equal to the number of branches (6) and n is equal to the number of buses of the power distribution system (3). The Y_{bus} matrix is equal to:

$$Y_{bus} = A_{branch}^{t} * Y_p * A_{branch}$$

In this case:

$$Y_{bus} = \begin{bmatrix} 0.0132-j2.2735 & -0.0083+j0.9094 & -0.0041+j0.4547 \\ -0.0083+j0.9094 & 0.0128+j0.1852 & 0 \\ -0.0041+j0.4547 & 0 & 0.0041-j0.4779 \end{bmatrix}$$

The incident matrix of the existing current sources is defined by:

$$A_{INL} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Each incident matrix element is 1 if the current is arriving to the bus or is –1 if the current leaves the respective bus, otherwise it is equal to zero. The dimension of A_{INL} is equal to the number of connected current sources (2) by the number of buses (3).

The system resonant frequencies can be found by plotting each element of H_{nodeI} as a function of the frequency (Fig. 2). The system transfer matrix, H_{nodeI}, in this case is equal to:

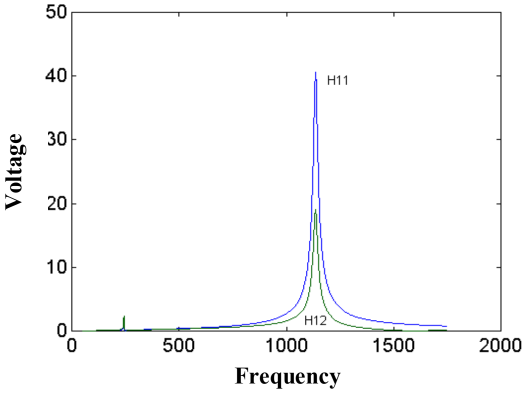
$$H_{nodeI} = Y_{bus}^{-1} * A_{INL}^{t}$$

$$H_{nodeI} = \begin{bmatrix} 0.01+j0.1586 & 0.0116-j0.7785 & 0.0095+j0.1509 \\ 0.017-j0.7785 & 0.017-j1.5761 & 0.0114-j0.7407 \\ 0.0095+j0.1509 & 0.0114-j0.7407 & 0.027+j2.2359 \end{bmatrix} * \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

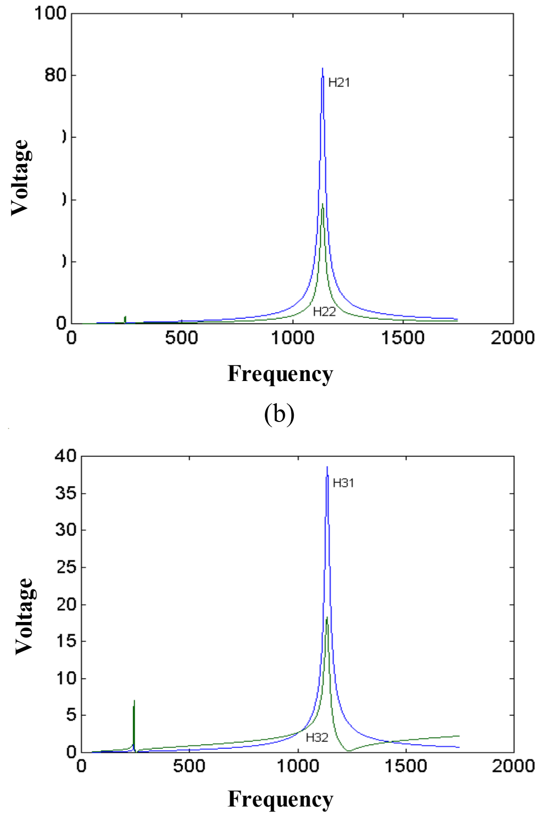
That is:

$$H_{nodeI} = \begin{bmatrix} 0.0116-j07785 & 0.0095+j0.1509 \\ 0.017-j1.5761 & 0.0114-j0.7407 \\ 0.0114-j0.7407 & 0.0271+j2.2359 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \end{bmatrix}$$

The values of the system current transfer matrix, H_{nodeI}, elements as a function of the frequency are shown in Fig. 2.



(a)



(b)

(c)

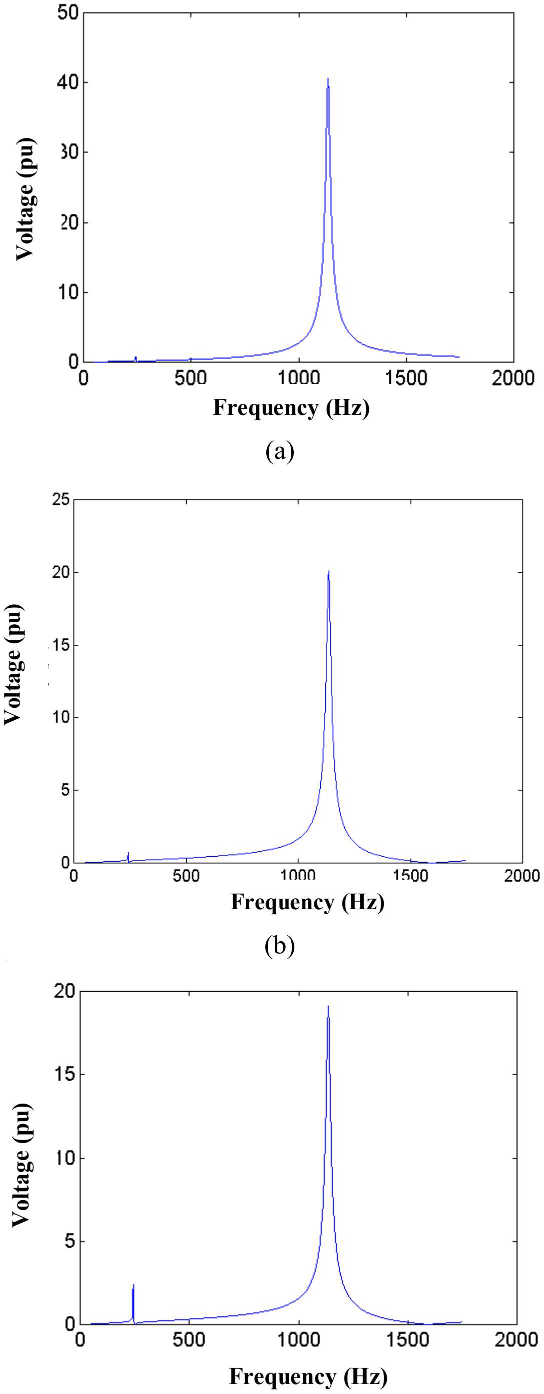
Fig. 2. Power distribution system impedance frequency response. (a) Frequency response for bus # 1. (b) Frequency response for bus # 2. (c) Frequency response for bus # 3.

The system voltage sensitive analysis allows finding the point of connection of the active power filter that produces the best voltage compensation in bus #1. This can be performed by simulating the connection of a shunt active power filter in each bus of the power distribution system, and finding the associated voltage distortion in each bus. In this case the incident matrix is equal to:

$$A_{INL}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

The 1 in the first row presents the active power filter when it is connected in bus #1, the 1 in the second row represents the active power filter when it is connected in bus # 2, while the 1 in row 3 represents the active power filter when it is connected in bus # 3. By multiplying the system impedance matrix Z_{bus} by the incident matrix A_{INL}^{t}, the system transfer matrix H_{nodeI} is obtained. By plotting the absolute values of each transfer matrix element as a function of the frequency the voltage sensitive analysis is derived.

In particular, Fig. 3 (a) shows the voltage response in bus # 1, when the active power filter is connected in the same bus. This figure shows that if a current equals 1 Ampere with a frequency of 1136 Hz is injected into bus # 1, a 20 per unit voltage in the same bus is generated. Figure 3 (b) shows that if 1 Ampere is injected in bus # 2 with the same frequency, the voltage induced in bus # 1 is 40 per unit, and finally, if 1 Ampere is injected in bus # 3, with the same frequency, the voltage induced in bus # 1 is only 19 per unit (Fig. 3 (c)).



(a)

(b)

(c)

Fig. 3. Voltage sensitivity analysis for bus # 1. (a) Voltage sensitivity response if the current is injected in bus # 1. (b) Voltage sensitivity response if the current is injected in bus # 2. (c) Voltage sensitivity response if the current is injected in bus # 3.

The voltage sensitivity analysis for bus # 1 shown in Fig. 3 concludes that the effectiveness of active compensation in the bus voltage #1 is significantly improved if the active power filter is connected in bus # 2, instead of connecting it in bus # 1 or bus # 3. In other words, by injecting current harmonics in bus # 2, the voltage distortion in bus # 1 is reduced more effectively than if the compensation is done in bus # 1 or in bus # 3. This result is correct since the bus # 2 injects more current harmonics to the power distribution system than the nonlinear load connected in bus # 3.

IV.- ACTIVE SHUNT COMPENSATION IN A MULTI-BUS INDUSTRIAL POWER SYSTEM

Figure 4 shows the single line diagram of an industrial power distribution system composed of large power nonlinear loads, motors and passive filters. The THD in current and voltages at the point of common coupling (bus # 1) exceeds the maximum values recommended by ANSI/IEEE Std. 519-1992. For this reason, it is necessary to connect a shunt active power filter that will help to reduce the THD in current and voltage at the PCC (bus #1). From the simple observation of the power distribution system shown in Fig. 4, the point of connection of the shunt active power filter cannot be derived. Moreover, the compensation effectiveness of the shunt active power filter can be severely affected if this compensator is not connected in the proper bus. By using the analytical procedure developed in Section III, the following power distribution operating characteristics are derived, so that the best connecting point of the shunt active power filter can be obtained.

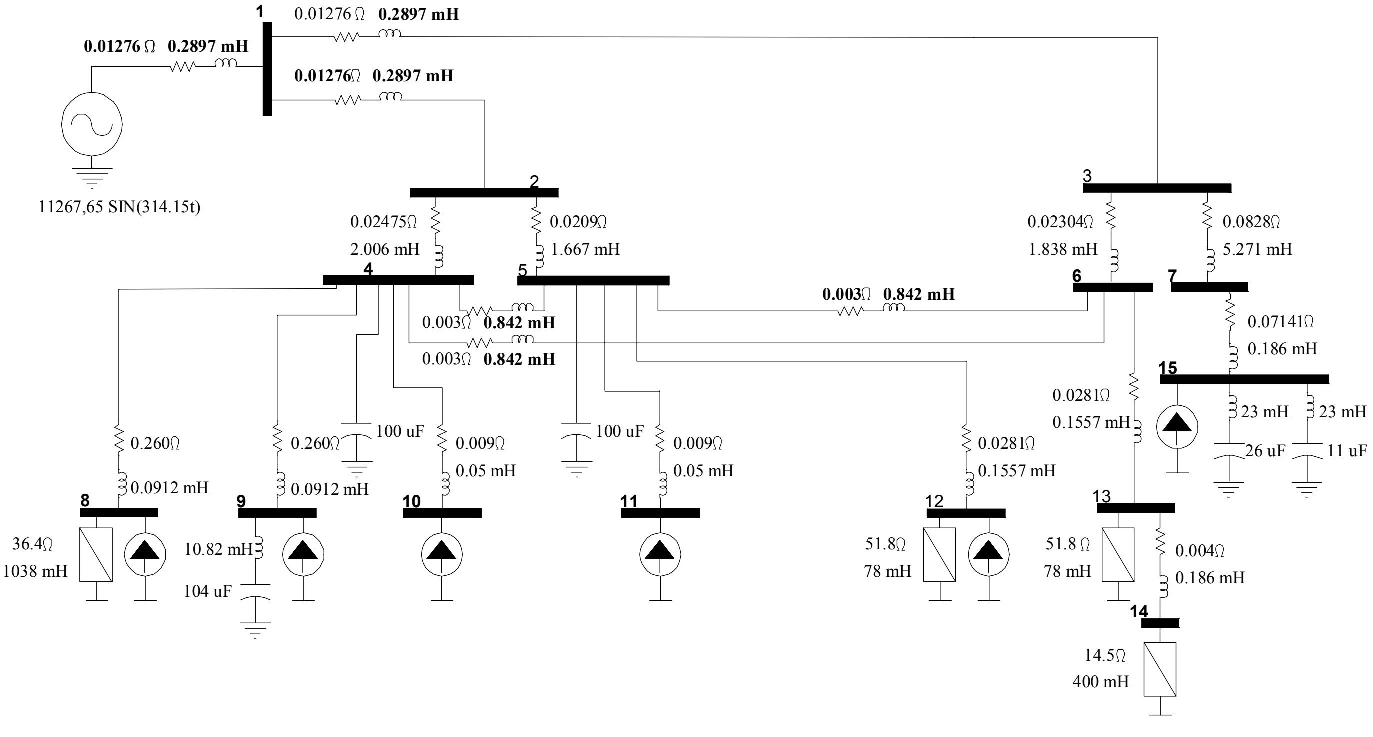


Fig. 4. Power distribution single line diagram of a steel mill plant.

4.1 Resonant frequencies

The resonant frequencies for different buses will be found (from the analysis of matrix H_{nodeI}) and the voltage sensitivity of bus #1 will be evaluated. Resonant frequencies activated by current injection in bus # 4, in different power distribution buses are shown in Fig. 5. It is clear that current injection in bus # 4 generates resonant frequencies at 1486 and 578 Hz. The voltage waveforms in buses 1, 2, 3, 6 and 7 are more affected when the resonant frequency is equal to 578 Hz. A more complete analysis proves that resonances do not increase voltages in the power distribution buses, making the system more reliable.

4.2 Voltage sensitivity analysis

This analysis allows finding the effect of current injection in different system buses and the voltage that is induced by these currents in a particular bus of the power distribution system. This result is very important to determine the best point of connection of an active power filter that will keep the voltage THD in bus # 1 below 3%. Moreover, once the best point of connection is defined, it is possible to evaluate the effect of the active compensation in all the other power system buses.

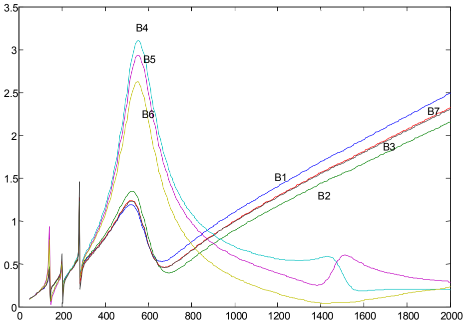


Fig. 5. Frequency response of the power distribution buses, for a current injected in bus # 4.

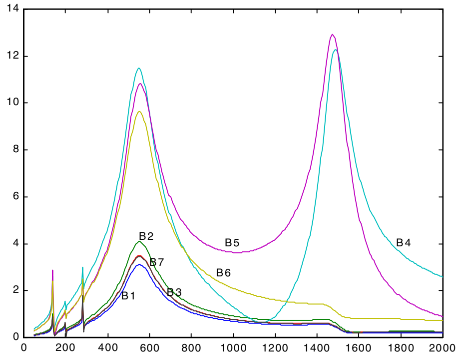


Fig. 6. Voltage sentivity analysis in bus # 1 for current injected in different buses.

The result of this analysis is shown in Fig.6, and illustrates the magnitude of each element of row 1 of the matrix H_{nodeI}. This figure shows that by connecting the shunt active power filter in bus # 4 the voltage distortion in all the other buses is improved, and the magnitude of the compensating current that is required to inject to the power distribution system is significantly reduced.

Table 1 shows the current amplitudes that need to be injected in each bus in order to have a voltage distortion equal to zero in bus #1. It is clear that by connecting the active power filter in bus # 4, the rms value of the active power filter current is reduced (1144 A). This result changes if the topology of the power distribution system is modified.

Table 1
Active filter current values for voltage compensation in bus # 1.

	I_{RMS}	h_1	h_5	h_7	h_{11}	h_{13}	h_{17}
Bus#1	1230	1071	309	249	307	329	65
Bus#2	1233	1073	309	242	267	366	86
Bus#3	1237	1072	310	251	293	359	74
Bus#4	1144	1075	297	197	109	87	59
Bus#5	1150	1073	309	205	116	93	66
Bus#6	1179	1075	306	210	129	111	93
Bus#7	1239	1071	310	255	296	362	75

The first column indicates the active power filter point of connection.

Also, Table 1 shows that if the active power filter is connected in bus # 1, a larger rms compensating current is required to keep the THDv in bus # 1 below 3 %, and an specially larger amplitude of current harmonic components is required to keep the low voltage distortion. A similar effect is obtained if the active power filter is connected in buses 2, 3, 5, 6 or 7. This result proves that the compensation effectiveness of the shunt active power filter depends on the point of connection.

Table 2 shows the effect in voltage distortion in the distribution system, for different point of connection of the active power filter. Again, the best point of connection is bus # 4, since by injecting the active power filter current in this bus, the voltage distortion in all the power system buses is reduced.

The same table shows that even though the connection of the active power filter in buses 1, 2 or 3, reduces THDv in these buses, it significantly increases the voltage distortion in buses 4, 5, and 6, due to resonances generated by the same active filter. Again, this result confirms the fact that compensation effectiveness in a multi-bus power system can be effectively improved by the proper selection of the active power filter point of connection.

Table 2
Voltage distortion (THD) in power distribution buses for different active power filter points of connection

	Active Power Filter Point of Connection						
	No Filter	Bus#1	Bus#2	Bus#3	Bus#4	Bus#5	Bus#6
Bus#1	3.92	0.00	0.00	0.00	0.00	0.00	0.00
Bus#2	5.15	1.51	0.59	1.47	0.00	0.00	0.16
Bus#3	4.40	0.64	0.58	1.47	0.00	0.00	0.16
Bus#4	14.22	12.57	0.25	0.27	0.03	0.03	0.03
Bus#5	13.35	11.69	10.61	11.39	1.13	1.22	1.27
Bus#6	12.02	9.99	9.09	9.35	0.06	0.06	2.55
Bus#7	4.36	0.64	0.58	1.46	0.00	0.00	0.16

V.- CONCLUSION

An analytical procedure that determines the more effective point of connection of shunt active power filters in multi-bus power distribution systems has been proposed. With this algorithm active current harmonics compensation performance can be optimized. A procedure based in the power distribution system transfer function matrices has been derived and carefully explained in a multi-bus industrial power distribution steel mill plant.

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