A New Mathematic Algorithm to Analyze Power Distribution Systems With Active Compensation and Nonlinear Loads

José Mahomar, Luis Morán, *Fellow, IEEE*, Johan Guzmán, *Member, IEEE*, and Juan Dixon, *Senior Member, IEEE*

***Abstract*—This paper presents a new procedure to analyze power distribution systems that energize nonlinear loads. The algorithm uses the singular values of the transfer matrix that relates the output variables and the system perturbation. The algorithm is especially developed to analyze the compensation performance of shunt and series active power filters. The algorithm is proved by simulation in a multibus industrial power distribution system and programmed in Matlab.**

***Index Terms*—Active power filter, harmonics, resonance.**

I. INTRODUCTION

ACTIVE power filters have proven to be an effective alternative to compensate current harmonics and reactive power in power distribution systems [1], [2]. Shunt active power filters operate as a controlled current source that follows a well-defined reference signal, allowing the current harmonic and reactive power compensation of a specific nonlinear load. On the other hand, series active power filters operate as controlled voltage sources and allow the operation of critical loads with constant balances and sinusoidal voltages. The compensation performance of such systems has always been proven in a simple way, by considering the Thévènin equivalent circuit referred to the bus where the nonlinear load is connected. In this scheme, the compensation performance of the shunt active power filter can be effectively demonstrated if the source current is sinusoidal and in phase with the respective phase-to-neutral voltage. However, in a real power distribution system, the compensation effectiveness can be affected depending of the point of connection of the active power filter [3]. Since shunt active power filters operate as controlled current sources generating a harmonic component, it is possible that the current generated by the shunt active power filter does not flow to the nonlinear load and propagate to the rest of the power distribution system, following the lowest impedance trajectory. This situation must be studied before using shunt active compensation in a multibus power distribution system, especially if other nonlinear loads are connected, shunt capacitors or tuned passive filters are used to compensate reactive power. Series compensation performance is proved if the voltage applied to the load is balanced, constant, and sinusoidal, independent of the perturbation introduced by the source. Again in this case, the performance of this type of compensation scheme in a multibus distribution system is not trivial, and the series active compensation can affect the adequate operation of the system by generating current harmonics, or by generating resonant frequencies.

Traditionally, power distribution systems that supply nonlinear loads are studied using power-flow analysis. This study is performed in the frequency domain, and gives the solution for steady-state operating conditions. If nonlinear loads or active compensation are connected to the power distribution system, harmonic analysis is performed by using superposition, and solving the power flow for each frequency component. This mathematical algorithm requires initial conditions and does not permit evaluating the system response for each type of perturbation, that is, the effect of each active compensator on the power distribution's voltages and currents.

The proposed algorithm is based on the nodal analysis of the power distribution system and the transfer matrix which relates the input to the output-power distribution system variables. The input variables are the branch voltages and the bus currents, while the output variables are the bus voltages and the branch currents. Simulated results obtained from a real multibus power distribution system prove the effectiveness of the proposed algorithm. The developed example helps to demonstrate the potential of this new mathematical tool, allowing to easily calculate resonant frequencies, total harmonic distortion, and compensation effectiveness of each connected active power filter.

A previously published paper, based on modal sensitivity analysis [9], [10] also used eigenvalues to evaluate resonant frequencies. However, these papers do not consider active compensation effectiveness and do not use the concept of the standard spectral. The method proposed in this paper allows improving active compensation in a power distribution system by selecting the active filter's best point of connection and calculating the resonant frequency in each power distribution bus.

II. BASIC EQUATION

The analysis of any power distribution system begins by writing the equations that relate the input with the output vari-

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J. Mahomar and J. Guzmán are with the Department of Electrical and Electronic Engineering, Universidad del Bío-Bío, Concepción, Chile (e-mail: jmahomar@ubiobio.cl).

L. Morán is with the Department of Electrical Engineering, Universidad de Concepción, Concepción, Chile (e-mail: lmoran@udec.cl).

J. Dixon is with the Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Santiago 7820436, Chile (e-mail: jdixon@ing.puc.cl).

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ables defining the transfer function and respective matrices. In this case, it is necessary to model the power distribution system and each active compensator or nonlinear load, depending on the type of study that needs to be done. The different models used for the power distribution systems and the series and shunt active power filters will be described.

A. Power Distribution System Model

The power distribution system is represented by the branch voltages and nodes currents, which are considered the input variables, and by the branch currents and nodes voltages, which are the output variables. The matrix that relates input and output variables is called the system transfer matrix. The transfer matrix is composed by four submatrices as shown in (1) with order $(\boldsymbol{b}+\boldsymbol{n}) \times (\boldsymbol{b}+\boldsymbol{n})$, where $\boldsymbol{b}$ is the number of system branches and $\boldsymbol{n}$ is the number of system nodes

$$[\boldsymbol{H}] = \begin{bmatrix} H_{VV} & H_{VI} \\ H_{IV} & H_{II} \end{bmatrix} [\boldsymbol{A}]^{\mathbf{U}}. \tag{1}$$

It is important to note that $[\boldsymbol{H}]$ is derived from the basic power-flow equation $[\boldsymbol{Y}_{\text{bus}}] \cdot [\boldsymbol{V}_{\text{bus}}] = [\boldsymbol{I}_{\text{bus}}]$. More details related to the derivation of this important equation can be found in [12], where $[\boldsymbol{A}]^{\mathrm{U}}$ is the connection matrix of the input variables and is defined as follows:

$$[\boldsymbol{A}]^U = \begin{bmatrix} A^{UV} & 0 \\ 0 & A^{UI} \end{bmatrix}. \tag{2}$$

A^{UV} and A^{UI} indicate how the voltage and current sources are connected, and by definition, are the system connection submatrices. A^{UV} has a number of rows that are equal to the power distribution system number of branches and a number of columns is equal to the number of independent voltage sources. In the same way, A^{UI} has a number of rows that are equal to the total number of nodes and columns which are equal to the number of independent current sources connected to the power distribution system. The dimension of the system connection matrix $[\boldsymbol{A}]^{\boldsymbol{U}}$ is $(\boldsymbol{b}+\boldsymbol{n}) \times 1$.

A^{UV} elements are 1, -1 or 0. The element is equal to 1 if the voltage polarity has the same direction of the respective current branch flow -1 if the voltage polarity and current flow are not equal and zero if the voltage source is not connected to the respective branch. As in the previous connection matrix, A^{UI} elements are again 1 -1 or 0. The element is equal to 1 if the current is arriving to the node, -1 if the current is leaving the node, and 0 if it does not exist. Input and output variables are defined as follows ($\mathbf{Y}$ output variables and $\mathbf{U}$ input variables):

$$\mathbf{Y} = \begin{bmatrix} V_n \\ I_b \end{bmatrix} \tag{3}$$

$$\mathbf{U} = \begin{bmatrix} V_b \\ I_n \end{bmatrix} \tag{4}$$

where $\boldsymbol{V}_n$ are the nodes voltages, $\boldsymbol{I}_b$ are the branch currents (system output variables), $\boldsymbol{V_b}$ are the branch voltages, and $\boldsymbol{I_n}$ are the nodes currents (system input variables).

The power distribution primitive matrix $[\boldsymbol{Z_p}]$ relates all of the power distribution branch impedances; it is a diagonal matrix order $\boldsymbol{b}\times\boldsymbol{b}$ (where b is the number of the power system branches) and the elements are equal to

$$\boldsymbol{Z}_{\text{pij}} = \begin{cases} 0 & \forall i \neq j \\ z_i & \forall i = j. \end{cases} \tag{5}$$

The branch connection matrix $[\boldsymbol{A}]$ gives information about the way the branches and nodes are connected. The elements are equal to 1 if the branch arrives at the node, -1 if the branch leaves the nodes, and 0 if they are not connected.

The output variables $\boldsymbol{Y}$ are related to the input variables $\boldsymbol{U}$ through the following equation:

$$[\boldsymbol{Y}] = [\boldsymbol{H}][\boldsymbol{U}] \tag{6}$$

where $[\boldsymbol{H}]$ is the transfer matrix defined in (1) and $[\boldsymbol{U}]$ represents the input variables defined in (4), and $[\boldsymbol{Y}]$ represents the output variables defined in (3).

The submatrices $\boldsymbol{H_{vv}}(\boldsymbol{n} \times \boldsymbol{b}), \boldsymbol{H_{vi}}(\boldsymbol{n} \times \boldsymbol{n}), \boldsymbol{H_{iv}}(\boldsymbol{b} \times \boldsymbol{b})$ and $\boldsymbol{H_{ii}}(\boldsymbol{b} \times \boldsymbol{n})$ are equal to

$$\boldsymbol{H_{VV}} = -\left[\mathbf{A}^{\mathrm{t}}\mathbf{Z}_{\mathrm{P}}^{-1}\mathbf{A}\right]^{-1}\mathbf{A}^{\mathrm{t}}\mathbf{Z}_{\mathrm{P}}^{-1} \tag{7}$$

$$\boldsymbol{H_{VI}} = \left[\mathbf{A}^{\mathrm{t}}\mathbf{Z}_{\mathrm{P}}^{-1}\mathbf{A}\right]^{-1} \tag{8}$$

$$\boldsymbol{H_{IV}} = -\left[\mathbf{Z}_{\mathrm{P}}^{-1}\mathbf{A}[\mathbf{A}^{\mathrm{t}}\mathbf{Z}_{\mathrm{P}}^{-1}\mathbf{A}]^{-1}\mathbf{A}^{\mathrm{t}} - \boldsymbol{I}\right]\mathbf{Z}_{\mathrm{P}}^{-1} \tag{9}$$

$$\boldsymbol{H_{II}} = \mathbf{Z}_{\mathrm{P}}^{-1}\mathbf{A}\left[\mathbf{A}^{\mathrm{t}}\mathbf{Z}_{\mathrm{P}}^{-1}\mathbf{A}\right]^{-1}. \tag{10}$$

The deduction of (7)–(10) is presented in [11]. Each submatrix represents the following:

- $\boldsymbol{H_{VV}}$ relates one input variable (branch voltage) with one output variable (node voltage);
- $\boldsymbol{H_{VI}}$ relates one input variable (node current) with one output variable (node voltage);
- $\boldsymbol{H_{IV}}$ relates one input variable (branch voltage) with one output variable (branch current);
- $\boldsymbol{H_{II}}$ relates one input variable (node current) with one output variable (branch current).

B. Active Power Filter Models

Active power filters have been developed to dynamically compensate power distribution systems. Series active power filters have been used to compensate voltage waveforms (distortion, unbalance, and amplitude), while shunt active power filters have been developed to compensate currents' distortion and load power factor. By compensating the distorted load current, ideally the respective bus voltage can be made perfectly sinusoidal. In the same way, by forcing the line current to be sinusoidal, using series compensation, the bus voltage is distorted by the voltage imposed by the series active power filter [14].

C. Shunt Active Power Filter Model

By definition, the shunt active power filter compensates current harmonics generated by nonlinear loads. If the shunt active power filter is connected to the same nonlinear bus, the

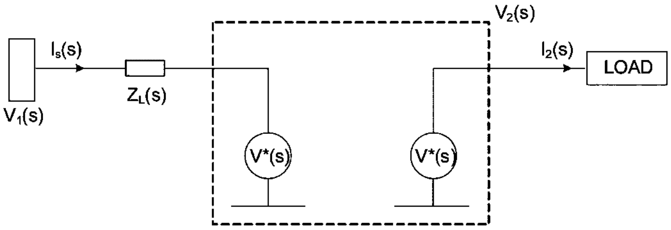

Fig. 1. Shunt active power filter equivalent circuit.

system line current is sinusoidal as well as the voltage. Therefore, the shunt active power filter, although it behaves as a current source in this case, from the power distribution point of view, can be modeled as a voltage source. In this case, the shunt active power filter forces the respective bus voltage to be sinusoidal. The equivalent circuit is shown in Fig. 1.

In this case, the final objective of the active shunt power filter is to keep the load-bus voltage sinusoidal, following a reference signal, therefore:

$$\mathrm{V}_2(\mathrm{s}) = \mathrm{V}*(\mathrm{s}). \tag{11}$$

In case the shunt active power filter is used to compensate only current harmonics, the only constraint imposed on $\boldsymbol{V}_{\mathrm{ref}}$ is that it must be sinusoidal. However, if the shunt active power filter is used to compensate current harmonics and power factor, $\boldsymbol{V}_{\mathrm{ref}}$ must be sinusoidal and in phase with the system line current $\boldsymbol{I_s}$ (Fig. 1).

D. Series Active Power Filter Model

The series active power filter can be used to compensate voltage regulation and unbalance as well as current harmonics. Current harmonics can be compensated using series active power filters by forcing the branch current to follow a given reference signal. If the reference signal is sinusoidal, the branch current will be sinusoidal, but the load-bus voltage will be distorted. Following this model, the series active power filter can be represented by an independent current source and a voltage source whose value and waveform depends on the load-bus voltage plus the voltage drop across the line impedance (12). The equation that models series compensation for harmonic components is shown in (12) as follows:

$$\boldsymbol{V_f^h}(\mathrm{s}) = \boldsymbol{V_1^h}(\mathrm{s}) - \mathbf{V}_2^{\mathrm{h}}(\mathrm{s}) - \boldsymbol{Z_L^h}(\mathrm{s})\boldsymbol{I}_s^h(\mathrm{s}). \tag{12}$$

In this case, series compensation is used to force the branch current to be sinusoidal or to follow a given reference current waveform; therefore, the branch current can be replaced by the given reference signal as follows:

$$\boldsymbol{I}_{\mathrm{s}}^{\mathrm{h}}(\mathrm{s}) = \boldsymbol{I}*(\mathrm{s}). \tag{13}$$

The equivalent circuit used to represent series active compensation for current harmonic elimination is shown in Fig. 2.

In this model, it is assumed that $\boldsymbol{V}_1(\boldsymbol{s})$ presents a THD below 3%.

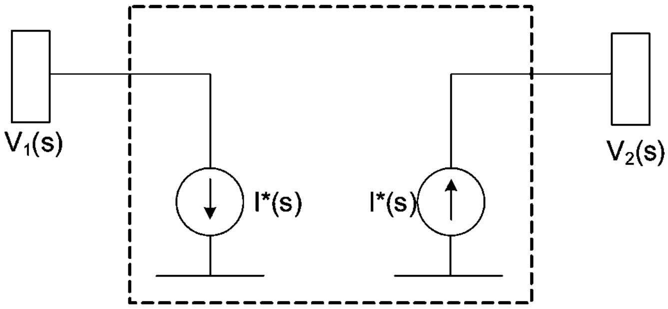

Fig. 2. Series active power filter equivalent circuit.

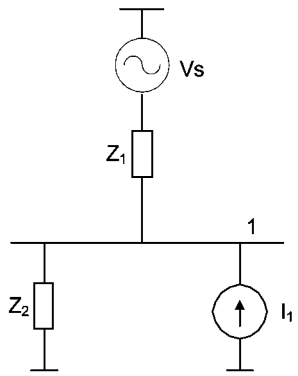

Fig. 3. Power distribution system equivalent circuit with a nonlinear load.

E. Output Variables Representation

If active power filters are used to compensate voltage or current waveforms, it is necessary to define the power distribution variables in order to evaluate the system response in front of the different perturbations; the original perturbations are represented by $\boldsymbol{U}^{\mathrm{P}}$, plus the reference signals used for active compensations, which are considered as independent input variables $\boldsymbol{U}^{\mathrm{R}}$. This representation allows analyzing the distribution system using two transfer matrices, and to obtain the system response for each applied perturbation. The system equations are the following:

$$\boldsymbol{Y} = \boldsymbol{G}^{\mathrm{P}}\boldsymbol{U}^{\mathrm{P}} + \boldsymbol{G}^{\mathrm{R}}\boldsymbol{U}^{\mathrm{R}} \tag{14}$$

where

$$\boldsymbol{U}^{\mathrm{P}} = \begin{bmatrix} V_r \\ I_b \end{bmatrix} \tag{15}$$

$$\boldsymbol{U}^{\mathrm{R}} = \begin{bmatrix} V_b^* \\ I_r^* \end{bmatrix}. \tag{16}$$

$\boldsymbol{G}^{\mathrm{P}}$ is the perturbation transfer matrix and $\boldsymbol{G}^{\mathrm{R}}$ is the reference transfer matrix. These matrices are equal to $\boldsymbol{G}^{\mathrm{P}} = [\mathbf{H}^{\mathrm{P}}][\mathbf{A}^{\mathrm{P}}]$ and $\boldsymbol{G}^{\mathrm{R}} = [\mathbf{H}^{\mathrm{R}}][\mathbf{A}^{\mathrm{R}}]$. The matrices $\boldsymbol{H}^{\mathrm{P}}$ and $\boldsymbol{H}^{\mathrm{R}}$ are obtained from (1) but considering the connections of the active power filters used to compensate the power distribution system. $\boldsymbol{A^P}$ and $\boldsymbol{A^R}$ are defined as the perturbation and refer-

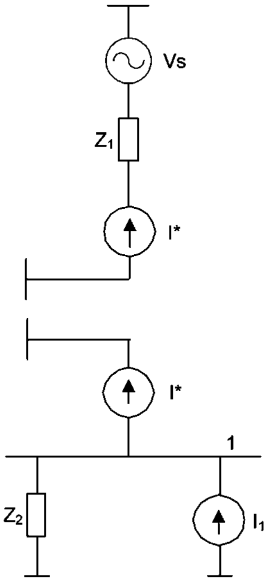

Fig. 4. Power distribution system equivalent circuit with series active compensation.

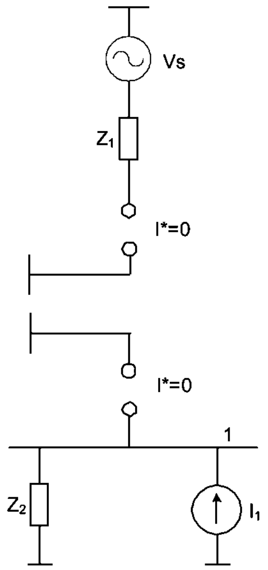

Fig. 5. Power distribution system equivalent circuit for perturbations.

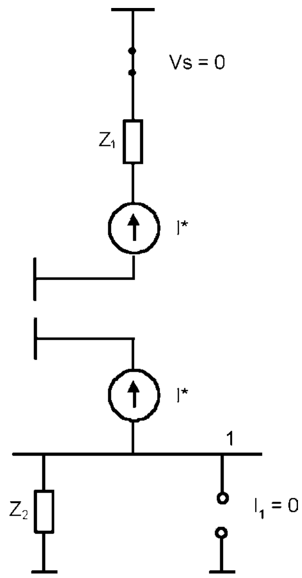

Fig. 6. Power distribution equivalent circuit derived to evaluate the reference signals effect.

ence connection matrices. Finally, the power distribution general equation is given by

$$\begin{aligned}[\boldsymbol{Y}] &= [\boldsymbol{G}^{\mathrm{P}}][\boldsymbol{U}^{\mathrm{P}}] + [\boldsymbol{G}^{\mathrm{R}}][\boldsymbol{U}^{\mathrm{R}}] \\ &= [\boldsymbol{H}^{\mathrm{P}}][\boldsymbol{A}^{\mathrm{P}}][\boldsymbol{U}^{\mathrm{P}}] + [\boldsymbol{H}^{\mathrm{R}}][\boldsymbol{A}^{\mathrm{R}}][\boldsymbol{U}^{\mathrm{R}}]. \end{aligned} \tag{17}$$

F. Power Distribution Model Example

The proposed methodology is explained with the following example shown in Fig. 3.

The example consists of a simple power distribution system composed with a nonlinear load, modeled as a current source, a passive impedance Z_2 as a linear load, and the power source (Vs) with the equivalent impedance Z_1 (Fig. 3). The nonlinear load injects a fundamental current component (0.93 p.u.) and a fifth current harmonic component (0.279 p.u.). Both currents are expressed over a 500-kVA base value. The different matrices associated with the power system example are as follows.

1) System connection matrix

$$A = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

2) Input connection matrix

$$A^U = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

3) Primitive impedance matrix

$$Z_P = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}$$

where $\boldsymbol{Z}_1 = $ j0.1 p.u. and $\mathrm{Z}_2 = -$j2.85 p.u.

4) Input vector variables

$$\mathbf{U} = \begin{bmatrix} V_s \\ I_1 \end{bmatrix}.$$

If a series active power filter is connected between the power source and bus 1, in order to compensate current distortion, the power system equivalent model is as shown in Fig. 4.

In order to obtain the different matrices that modeled the compensated power distribution system, the equivalent circuits shown in Figs. 5 and 6 are derived.

Fig. 5 shows the system equivalent circuit used to obtain the transfer matrix and the associated input vector. By using series compensation, the power distribution system is decoupled from the power supply, and the nonlinear current source is connected in parallel to system impedance Z_2.

The different matrices associated with the perturbations are as follows.

1) New primitive impedance matrix

$$Z_P = [Z_2].$$

2) New system connection matrix

$$A = [1].$$

3) Perturbation connection matrix

$$A^P = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

4) Input vector variables

$$U^P = [I_1].$$

In order to evaluate the effect of the reference signals in the output variables, the equivalent circuit shown in Fig. 6 is derived. In this equivalent circuit, the source voltage (VS) and source current (I1) are equal to zero.

Fig. 6 shows that branch 1 is not considered since the associated current does not depend on Z1; therefore, only Z2 is connected to the main nodes. From the equivalent circuit shown in Fig. 6, the system transfer matrix and the reference vector are derived.

1) System primitive impedance matrix

$$Z_P = [z_2].$$

2) System connection matrix

$$A = [1].$$

3) Reference connection matrix A^R

$$A^R = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

4) Input vector variables (associated with the references) UR

$$U^R = [I^*].$$

The reference signal I$*$ is defined by the series active power filter control scheme in order to obtain sinusoidal current with unity power factor. The power distribution output variables can be obtained by using superposition since the perturbation and reference signals are considered in this analysis. The system output variable vector is

$$[Y] = \begin{bmatrix} Vb_1 \\ Ir_1 \\ Ir_2 \end{bmatrix}.$$

The system output variables are calculated for each harmonic frequency present in the perturbation signals. The final results obtained with this technique are shown in Figs. 7–13.

III. System Transfer Matrix

The principal objective of this paper is to develop a mathematical tool capable of showing power distribution characteristics, especially when nonlinear loads are connecting. These characteristics are related to the power distribution frequency response, voltage and current distortion in different buses, bandwidth at specific points, resonant frequencies, and impedance versus frequency response. All of this information can be deployed from the power distribution transfer matrix.

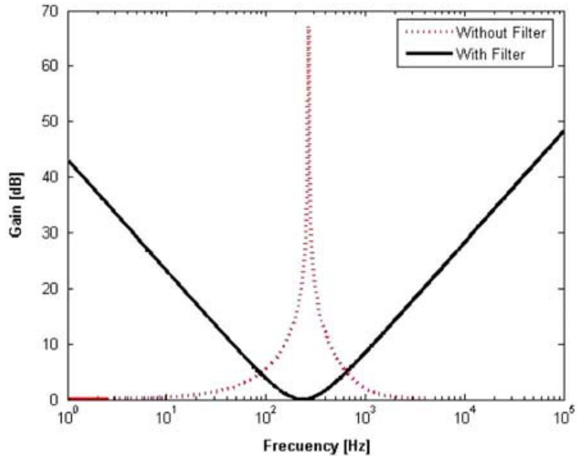

Fig. 7. System voltage gain as a function of the frequency, with active series and without active series compensation.

A. Power Distribution Transfer Matrix Evaluation

It is important to develop an easy and fast way to find out the possible changes introduced in the resonant frequencies defined by $\mathbf{G}^{\mathrm{P}}$ due to nonlinear loads. One way of doing it is to analyze the system transfer function referring to each bus one by one, or another possibility is to evaluate the magnitude of $\mathbf{G}^{\mathrm{P}}$, corresponding to the highest singular value. The singular values of a matrix $\mathbf{H}$ are equal to the square root of the first $\boldsymbol{k}$ proper values of its Hermitean, as is defined in [13]

$$\sigma_i(s) = \sqrt{\lambda_i(H(s)^H H(s))}. \quad (18)$$

The gain of a given input variable $\boldsymbol{u}$ is defined as the ratio between the magnitude of $\mathbf{H}$ in the direction of $\boldsymbol{u}$ and the magnitude of $\boldsymbol{u}$. It is important to note that for this definition, the gain cannot be higher than maximum singular values (19)

$$\min(\sigma(H)) < \left.\frac{||Hu||_2}{||u||_2}\right|_{u\neq 0} < \max(\sigma(H)). \quad (19)$$

The analysis of $\mathbf{G}^{\mathrm{P}}$ gives information regarding resonant frequencies at a particular bus, especially if the system gain increases at a particular frequency, or if the resonant frequency associated with a large gain changes. Both cases can be the source of important power distribution operation problems, and are related to the maximum singular value. In other words, by analyzing the magnitude of $\mathbf{G}^{\mathrm{P}}$, resonant problems associated with the connection of passive or active filters can be found.

IV. Example

Fig. 14 shows the single-phase equivalent circuit of an industrial power distribution system. Power distribution system parameters are shown in Fig. 15. Nonlinear loads are represented by current sources and typical linear loads by the equivalent

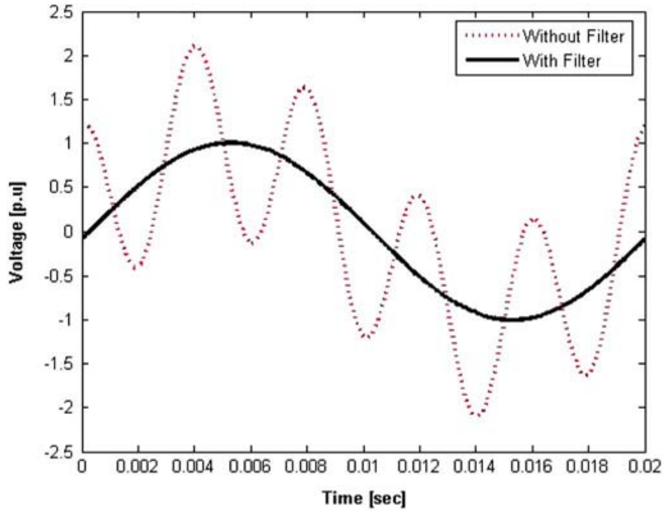

Fig. 8. Voltage waveform in bus 1, with and without the filter.

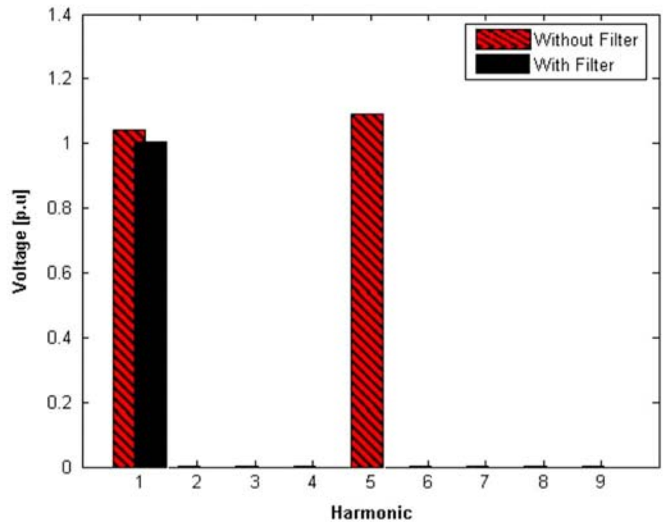

Fig. 9. Voltage frequency spectrum of bus 1, with and without the filter.

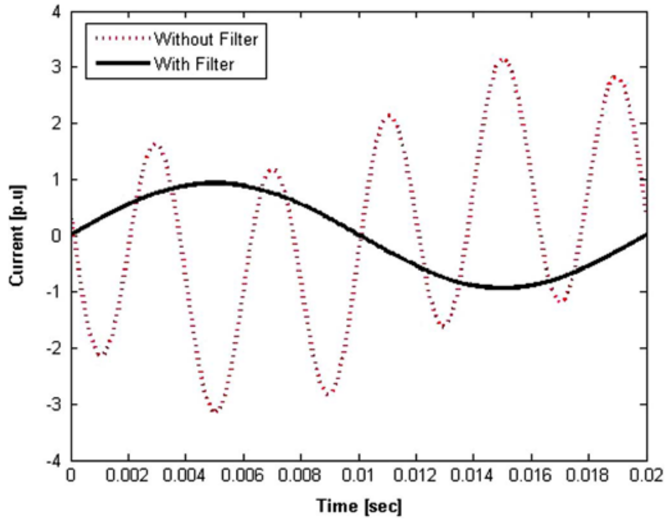

Fig. 10. Current waveform of branch 1, with and without the filter.

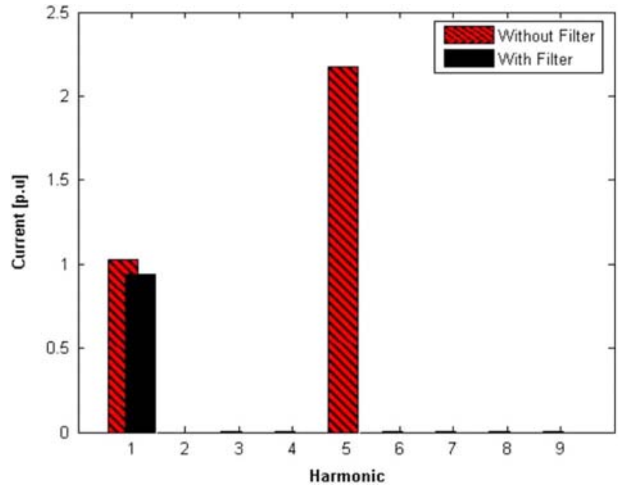

Fig. 11. Branch 1 current frequency spectrum, with and without the filter.

RL circuit (Fig. 16). In this case, the shunt and series active power filter will be connected in order to reduce voltage and current total harmonic distortion at the point of common coupling (PCC), and particular internal buses (Fig. 16). Power distribution system equivalent circuits for perturbation and for reference effect analyses are shown in Figs. 17 and 18.

1) New primitive impedance matrix (see the equation at the bottom of the page)

$$Zp = \begin{bmatrix} z_2 & 0 & \cdot & \cdot & \cdot & & & & \cdot & \cdot & \cdot & 0 \\ 0 & z_3 & \cdot & & & & & & & & & \\ \cdot & \cdot & z_4 & \cdot & & & & & & & & \\ \cdot & & \cdot & z_5 & \cdot & & & & & & & \\ \cdot & & & \cdot & z_6 & & & & & & & \\ & & & & & z_7 & & & & & & \\ & & & & & & z_8 & & & & & \\ & & & & & & & z_{11} & \cdot & & & \cdot \\ \cdot & & & & & & & \cdot & z_{12} & \cdot & & \cdot \\ \cdot & & & & & & & & \cdot & z_{13} & \cdot & \cdot \\ \cdot & & & & & & & & & \cdot & z_{14} & 0 \\ 0 & & & & & & & \cdot & \cdot & \cdot & 0 & z_{15} \end{bmatrix}.$$

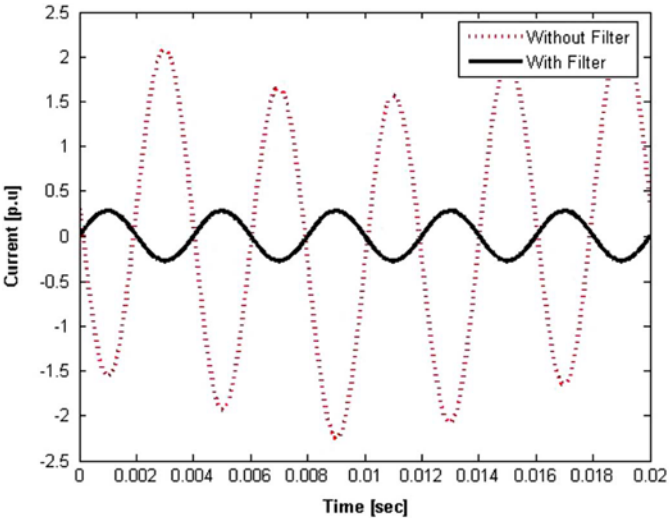

Fig. 12. Branch 2 current waveform with and without the filter.

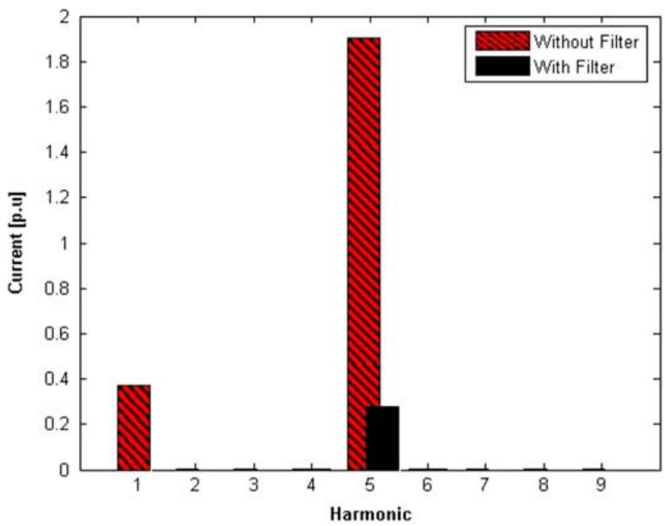

Fig. 13. Branch 2 current frequency spectrum, with and without the filter.

2) New system connection matrix

$$
A=\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0\\
1 & 0 & -1 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & -1 & 0 & 0\\
0 & 0 & 1 & 0 & -1 & 0\\
0 & 0 & 1 & 0 & 0 & -1\\
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
$$

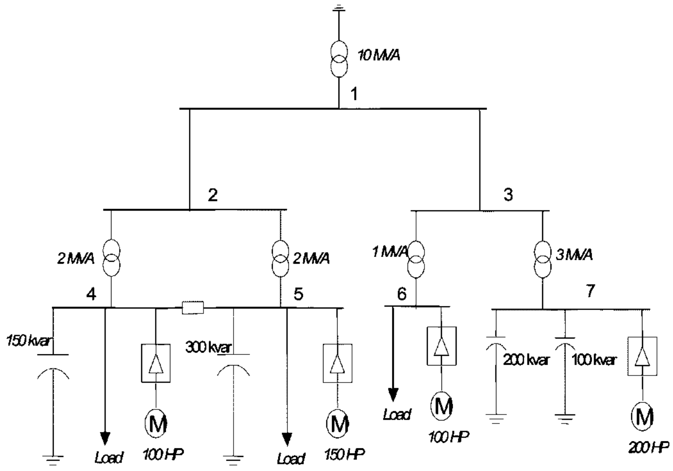

Fig. 14. Industrial power distribution system equivalent diagram.

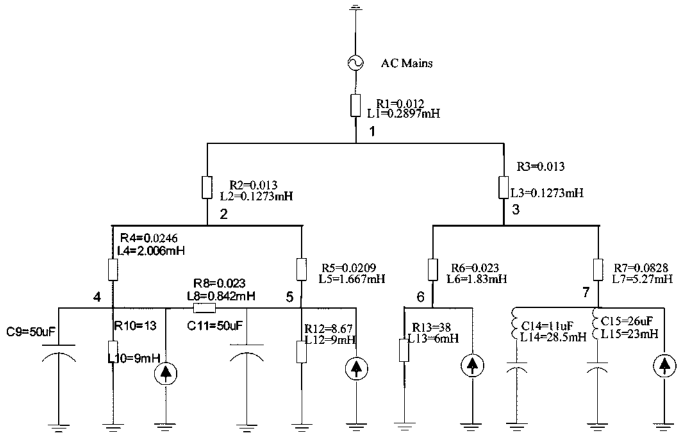

Fig. 15. Industrial power distribution system single-phase equivalent circuit.

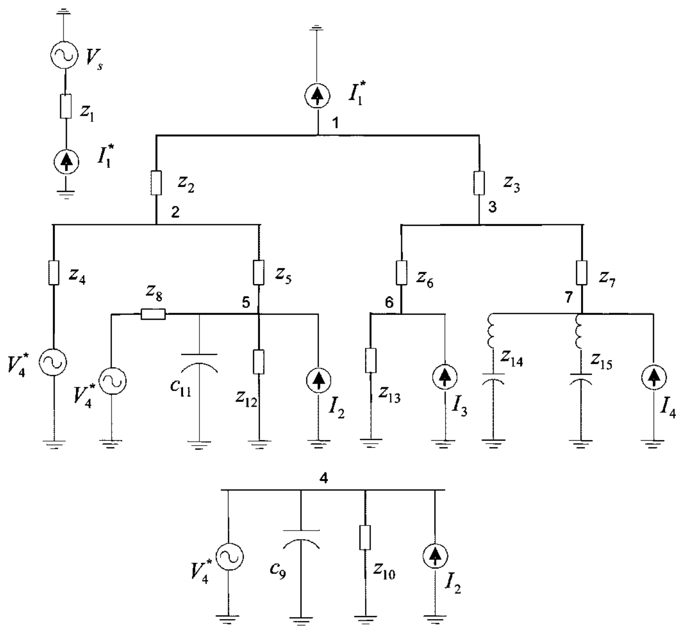

Fig. 16. Power distribution system equivalent circuit with series and shunt active compensation.

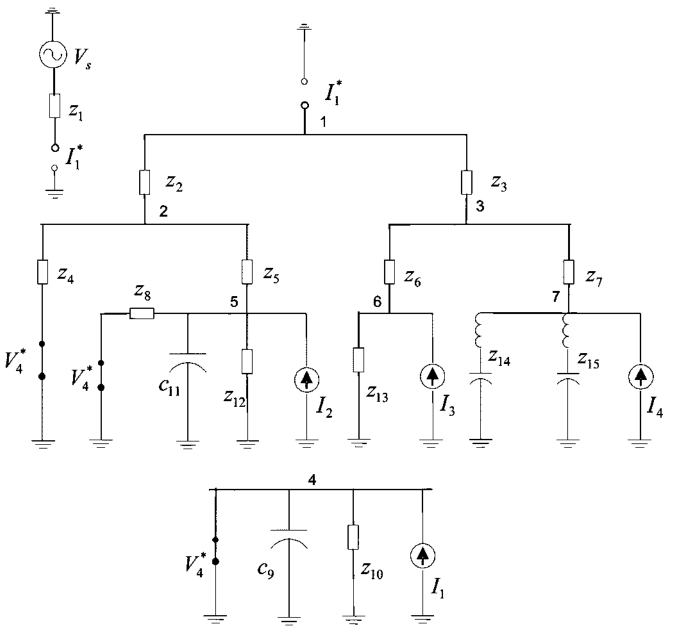

Fig. 17. Power distribution system equivalent circuit for perturbations.

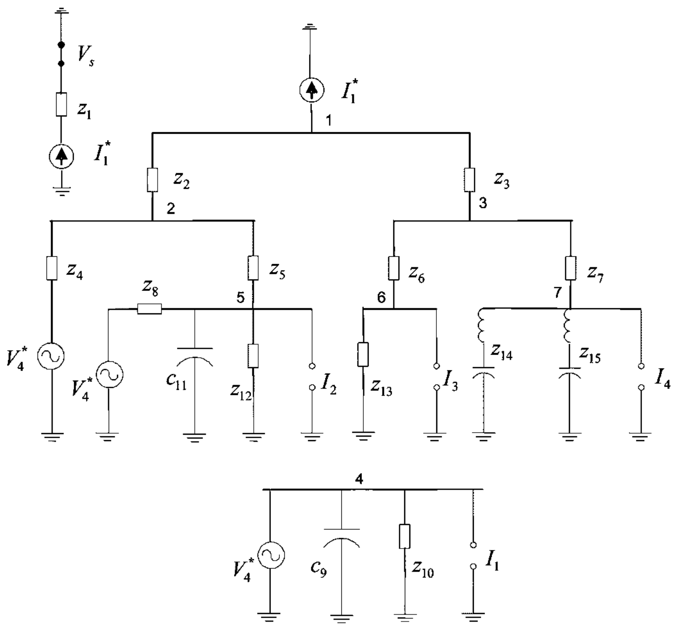

Fig. 18. Power distribution equivalent circuit for reference effect analysis.

3) Perturbation connection matrix

$$A^P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4) Input vector variables

$$U^P = \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix}.$$

Changes in system resonant frequency, due to the connection of passive or active filters, can be obtained from the analysis of GP. In this particular case, by connecting a shunt active power filter in bus 4, the interaction between the capacitor

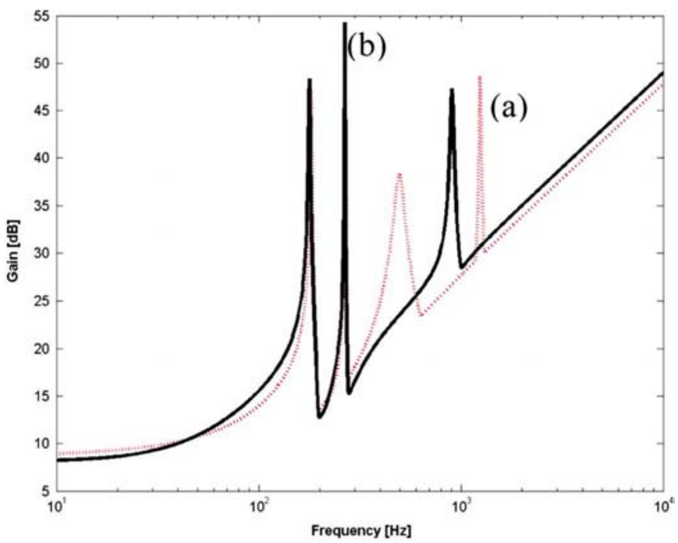

Fig. 19. Maximum singular values for G^P. (a) Without active power compensation. (b) With active compensation.

connected in bus 4 and the power distribution system is eliminated, avoiding the generation of resonance. The analysis of the system response due to the current injection in buses 5, 6, and 7, as shown in Figs. 19–22 indicates that the power distribution system did not change, but Fig. 20 shows an increase the current gain of branches 4, 5, 8, and 11 due to the resonance generated between the capacitor connected in bus 5 and the power distribution system.

V. Conclusion

A new procedure to analyze power distribution systems that energize nonlinear loads based on the power distribution transfer matrix was proposed in this paper. The algorithm uses the singular values of the transfer matrix that relates the output variables and the system perturbation. The algorithm is

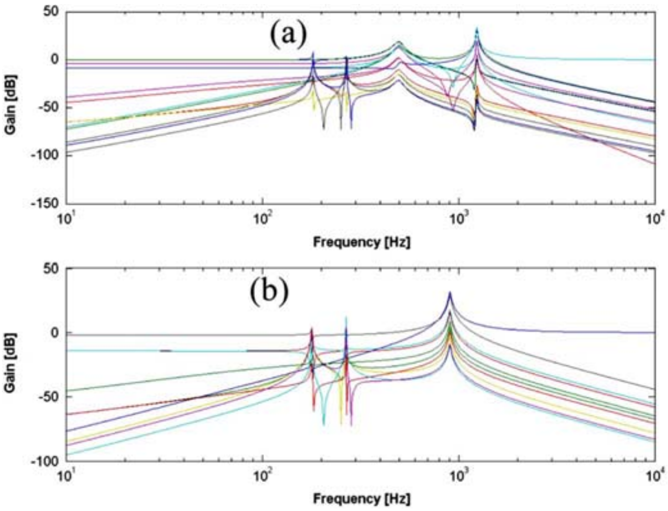

Fig. 20. System transfer matrix for branch currents with respect to the current injected in branch 5. (a) Without active compensation. (b) With active compensation.

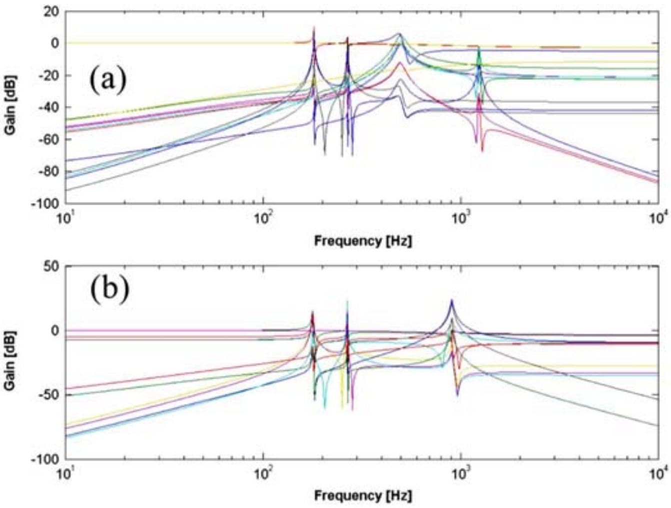

Fig. 21. System transfer matrix for branch currents with respect to the current injected in branch 6. (a) Without active compensation. (b) With active compensation.

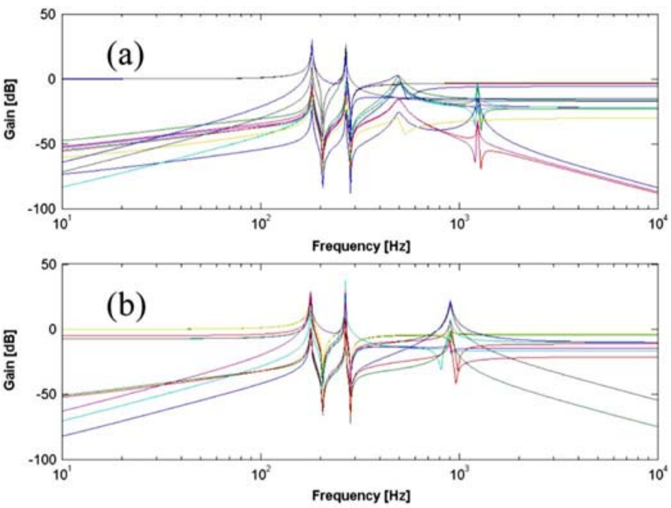

Fig. 22. System transfer matrix for branch currents with respect to the current injected in branch 7. (a) Without active compensation. (b) With active compensation.

especially developed to analyze the compensation performance of shunt and series active power filters. The algorithm was proved by simulation in a multibus industrial power distribution system.

REFERENCES

[1] D. Andrews, M. T. Bishop, and J. F. Witte, "Harmonic measurement, analysis, and power factor correction in a modern steel manufacturing facility," *IEEE Trans. Ind. Appl.*, vol. 32, no. 3, pp. 617–624, May/Jun. 1996.

[2] P. Y. Keskar, "Specification of variable frequency drive systems to meet the new IEEE 519 standard," *IEEE Trans. Ind. Appl.*, vol. 32, no. 2, pp. 393–402, Mar./Apr. 1996.

[3] R. L. Almonte and W. Ashley, "Harmonic at the utility industrial interface: A real world example," *IEEE Trans. Ind. Appl.*, vol. 31, no. 6, pp. 1419–1425, Nov./Dec. 1995.

[4] J. P. Nelson, "A better understanding of harmonic distortions in the petrochemical industry," *IEEE Trans. Ind. Appl.*, vol. 40, no. 1, pp. 220–231, Jan./Feb. 2004.

[5] W. M. Grady and S. Santoso, "Understanding power system harmonic," *IEEE Power Eng. Rev.*, vol. 21, no. 11, pp. 8–11, Nov. 2001.

[6] P. C. Buddingh, "Even harmonic resonance—An unusual problem," *IEEE Trans. Ind. Appl.*, vol. 39, no. 4, pp. 1181–1186, Jul./Aug. 2003.

[7] E. J. Currence, J. E. Plizga, and H. N. Nelson, "Harmonic resonance at a medium-sized industrial plant," *IEEE Trans. Ind. Appl.*, vol. 31, no. 4, pp. 682–690, Jul./Aug. 1995.

[8] A. F. Zobaa, "Cost-effective applications of power factor correction for nonlinear loads," *IEEE Trans. Power Del.*, vol. 20, no. 1, pp. 359–365, Jan. 2005.

[9] Z. Huang, Y. Cui, and W. Xu, "Application of modal sensitivity for power system harmonic resonances analysis," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 222–231, Feb. 2007.

[10] Y. Cui and W. Xu, "Harmonic resonance mode analysis using real symmetrical nodal matrices," *IEEE Trans. Power Del.*, vol. 22, no. 3, pp. 1989–1990, Jul. 2007.

[11] L. Morán, J. Mahomar, and J. Dixon, "Selecting the best point of connection for shunt active power filters in multibus power distribution systems," *IEEE Ind. Appl. Mag.*, vol. 10, no. 2, pp. 43–50, Mar./Apr. 2004.

[12] T. Gönen, *Modern Power System Analysis*. New York: Wiley, 1988.

[13] K. Ogata, *Modern Control Engineering*, 4th ed. Englewood Cliffs, NJ: Prentice-Hall.

[14] B. Singh, K. Al-Haddad, and A. Chandra, "A review of active filter for power quality improvement," *IEEE Trans. Ind. Electron.*, vol. 46, no. 5, pp. 960–971, Oct. 1999.

José Mahomar was born in Concepción, Chile, in 1962. He received the B.S. and M.Sc. degrees in electrical engineering from the University of Concepción, Concepción, Chile, in 1987 and 2002, respectively, where he is currently pursuing the Ph.D. degree in electrical engineering.

He is a Professor with the Department of Electrical and Electronic Engineering, Universidad del Bio-Bio, Concepción, where he has been since 1989. His research interests include power quality and active power filters.

Luis Morán (F'05) was born in Concepción, Chile. He received the electrical engineering degree from the University of Concepción, Concepción, Chile, in 1982, and the Ph.D. degree from Concordia University, Montreal, QC, Canada, in 1990.

Since 1990, he has been with the Electrical Engineering Department, University of Concepción, where he is a Professor. He has written and published more than 50 papers in active power filters and static var compensators in IEEE TRANSACTIONS. He has extensive consulting experience in the mining industry, especially in the application of medium-voltage ac drives, large power cycloconverter drives for SAG mills, and power-quality issues. His main areas of interests are in Ac drives, power quality, active power filters, flexible ac transmission systems, and power protection systems.

Dr. Morán is the principal author of the paper that received the IEEE Outstanding Paper Award from the Industrial Electronics Society for the best paper published in the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS during 1995, and the coauthor of the paper that was awarded by the IAS Static Power Converter Committee in 2002. In 1998, he received the City of Concepción Medal of Honor for achievement in applied research. From 1997 to 2001, he was Associate Editor of the IEEE TRANSACTIONS ON POWER ELECTRONICS.

Johan Guzmán was born in Lota, Chile, in 1976. He received the M.Sc. degree in electrical engineering from the University of Concepción, Concepción, Chile, in 2003, where he is currently pursuing the Ph.D. degree.

His research areas include modulation and control of large ac–dc and dc–ac converters and renewable energies.

Juan Dixon (SM'95) was born in Santiago, Chile. He received the Ms.Eng. and the Ph.D. degrees from McGill University, Montreal, QC, Canada in 1986 and 1988, respectively.

In 1976 he was with the State Transportation Company in charge of trolleybus operation. In 1977 and 1978, he was with the Chilean Railways Company. Currently, he is a Professor with the Electrical Engineering Department, Pontificia Universidad Catolica de Chile, where he has been since 1979. He has published more than 25 papers related to Power Electronics in the IEEE TRANSACTIONS and IEE Proceedings. His main areas of interest are electric traction, power converters, pulsewidth-modulated rectifiers, active power filters, power factor compensators, and multilevel and multistage converters. He has consulting work related to trolleybuses, traction substations, machine drives, hybrid electric vehicles, and electric railways. He has created an Electric Vehicle Laboratory, where he has built state-of-the-art vehicles using brushless-dc machines with ultracapacitors and high-specific energy batteries. Recently, he has begun research in distributed generation and power generation using renewable energy sources.