A Simple Frequency-Independent Method for Calculating the Reactive and Harmonic Current in a Nonlinear Load

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***Abstract*— A basic criterion that determines the behavior of an active power filter is the method of calculating the reference current. There are many ways of generating this reference, but the methods are generally complex and hard to tune. This paper describes a simple and effective method for calculating the reference current, necessary to feed a shunt active power filter to compensate the power factor and harmonic currents generated by a nonlinear load. Simulations and experimental results are presented, showing that the proposed circuit may operate at frequencies ranging from 40 to 65 Hz without adjustment.**

I. Introduction

Harmonic contamination in power systems is a serious problem due to the increase of nonlinear loads over the last 20 years, such as static power converters, arc furnaces, and others.

The harmonic problem is not solved adequately by passive LC filters due to their inability to compensate random frequency variations in the currents, tuning problems, and parallel resonance. To overcome these problems, active power filters have been developed, such as the shunt active filter shown in Fig. 1, one of the most common topologies [1], [2]. In this figure, a full-wave rectifier with a dc filter inductor has been added as an example of harmonic contaminating load.

The shunt active power filter is a voltage source inverter controlled as a current source by means of a pulsewidth modulation (PWM) signal. As it can be seen in Fig. 1, the shunt active filter is connected in parallel with the load that is being compensated. If the filter generates the harmonic currents I_F that are required by the load, the mains supply delivers only the fundamental current I_S and the harmonics are eliminated from the power lines. Adequate control of the active filter not only eliminates the power line harmonics, but can also improve power factor.

The performance of these active filters is based on three basic design criteria [3], [8]:

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1) inverter parameters;
2) PWM control method used;
3) method used to obtain the current reference, which is the subject of this paper.

Traditional methods used to obtain the reference current required by the active filter are based on electronic tuned filters and instantaneous power theory [4]–[6]. Electronic filters, usually of the bandpass type, have the disadvantage that a small change in the mains frequency may cause significant phase shift at the output. This forces the use of precision components and the need for frequent adjustment. Even if it were possible to trim components to tune the circuit, this would be valid only for a specific operating frequency and would not compensate variations due to component aging and temperature. On the other hand, the instantaneous power theory requires complex circuitry to implement the transformations, usually from four to six high-precision analog multipliers per phase. This makes the circuit complex and sensitive to component parameter variations.

A new circuit that is simpler than the instantaneous power theory-based design will be presented. This circuit requires no adjustments at all and can work properly in the continuous range of frequencies from 40 to 65 Hz, covering both 50- and 60-Hz distribution systems. The proposed circuit permits both harmonic current cancellation and power factor improvement.

II. General Description of the Active System

A. Basic Principle

Without active filter compensation, the line current, which is distorted by the power factor and harmonics of the characteristics of the load, results in a load current I_L made up of the following four terms:

$$i_L(t) = i_o(t) + i_p(t) + i_q(t) + i_h(t) \tag{1}$$

where

i_o dc component;
i_p in-phase line current;
i_q reactive current;
i_h harmonic currents.

Equation (1) can be further expanded as shown in (2), where the first and second summation represent the even and odd harmonics, respectively. This is a very general form

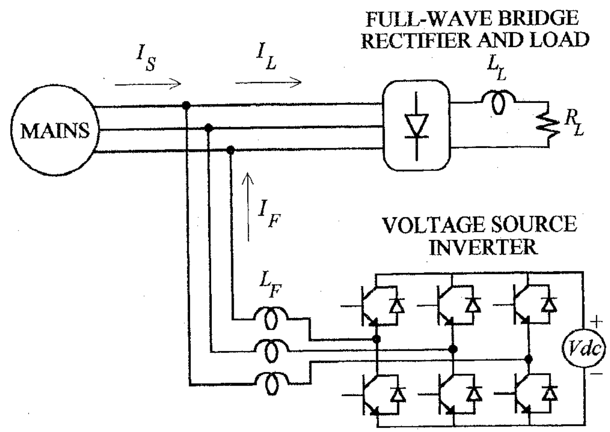

Fig. 1. Voltage source inverter as a shunt active filter.

of load current. However, in practice, the dc component is usually small or it does not exist at all. Also, when no neutral connection is used zero sequence harmonics do not exist

$$\begin{aligned} i_L(t) = I_o &+ I_p\cos(\omega t) + I_q\sin(\omega t) \\ &+ \sum_{j=1}^{\infty} I_{2j}\cos(2j\omega t + \phi_{2j}) \\ &+ \sum_{k=1}^{\infty} I_{2k+1}\cos((2k+1)\omega t + \phi_{2k+1}). \qquad (2) \end{aligned}$$

The only component that the mains should supply is the active current $i_p[I_p\cos(\omega t)]$. Using (1), it can be noted that if the active filter supplies the dc component, the reactive and the harmonic currents for the load, then the mains needs only to supply the active current. This can be easily accomplished by subtracting the active current component i_p from the measured load current i_L

$$i_F(t) = i_L(t) - i_p(t) = i_L(t) - I_p\cos(\omega t). \qquad (3)$$

In (3), I_p is the magnitude of the in-phase current (which needs to be estimated) and $\cos(\omega t)$ is a sinusoid in phase with the line voltage. This operation can be accomplished by the circuit shown in Fig. 2. The estimation of I_p is now explained. Let us consider the product of the load current of (2) and a sinusoid in phase with the line voltage

$$\begin{aligned} i_L(t)\cdot\cos(\omega t) = I_o\cos(\omega t) &+ \frac{I_p}{2}[1+\cos(2\omega t)] \\ &+ \frac{I_q}{2}\sin(2\omega t) \\ &+ \sum_{j=1}^{\infty}\frac{I_{2j}}{2}[\cos((2j+1)\omega t + \phi_{2j}) \\ &\qquad + \cos((2j-1)\omega t + \phi_{2j})] \\ &+ \sum_{k=1}^{\infty}\frac{I_{2k+1}}{2}[\cos((2k+2)\omega t + \phi_{2k+1}) \\ &\qquad + \cos(2k\omega t + \phi_{2k+1})]. \qquad (4) \end{aligned}$$

After the multiplication, the only dc term present in (4) is proportional to I_p. Thus, a low-pass filter whose cutoff

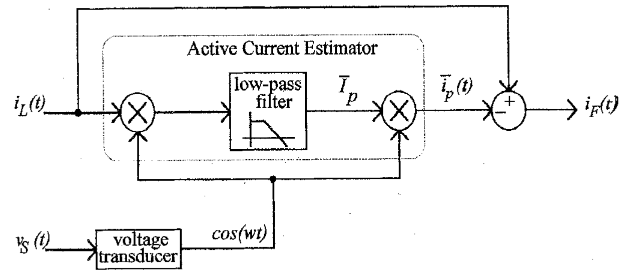

Fig. 2. Open-loop diagram for calculating the compensating current (one phase).

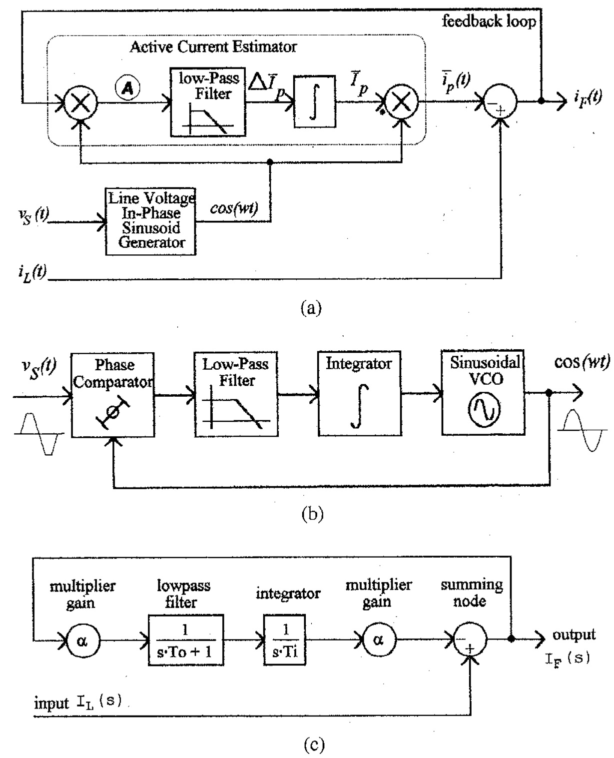

Fig. 3. Closed-loop diagram for calculating the compensating current (one phase). (a) The proposed circuit. (b) In-phase sinusoid generator. (c) Simplified block diagram in Laplace domain.

frequency is below ω permits one to obtain $\bar{I}_p$, which is an estimation of the magnitude of $i_p(t)$. Then, this dc value is multiplied by the same in-phase sinusoid, obtaining an estimation of the instantaneous active current $\bar{i}_p(t)$. Finally, this value of $\bar{i}_p(t)$ is subtracted from the measured load current $i_L(t)$ obtaining the required reference current $i_F(t)$.

The aforementioned method of obtaining the in-phase current, whose diagram is shown in Fig. 2, is an open-loop implementation, and presents the following problems.

1) The amplitude of the in-phase fundamental reference voltage, scaling factors of the analog multipliers, and the low-pass filter gain affect the magnitude of the active current estimation $\bar{i}_p(t)$.
2) A small phase shift in the voltage transducer will degrade the accuracy of the circuit.

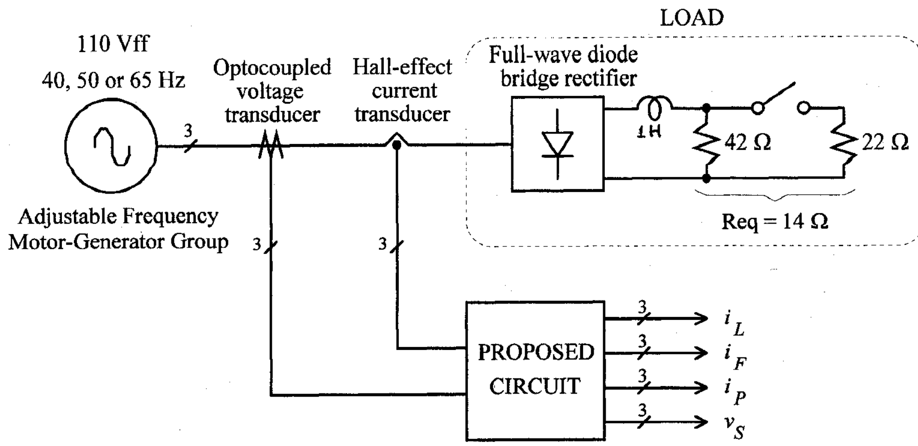

Fig. 4. Experimental circuit with a rectifier load.

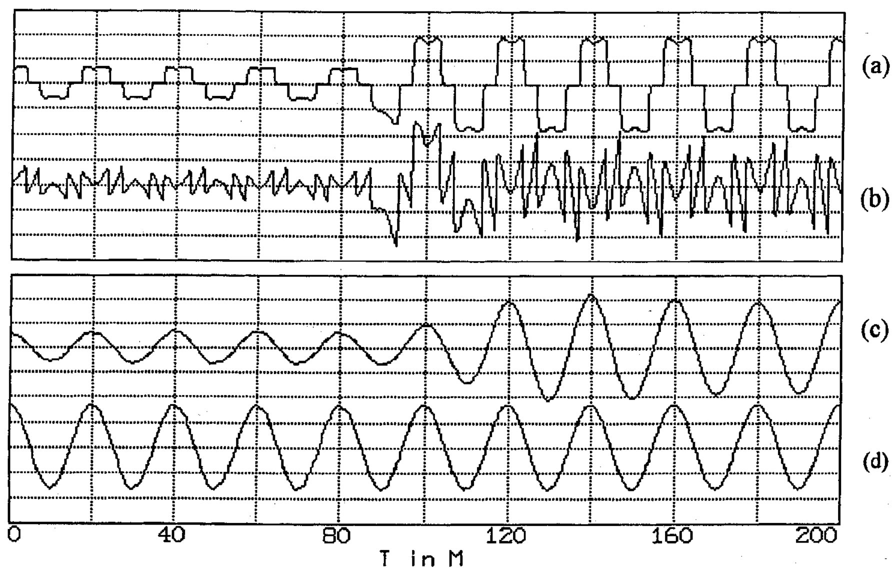

Fig. 5. Simulation with a rectifier load (50 Hz). (a) Load current $i_L(t)$, 5 A/div. (b) Calculated compensating current $i_F(t)$, 2.5 A/div. (c) Estimated active current $i_p(t)$, 5 A/div. (d) Line voltage reference $v_L(t)$, 5 V/div.

3) Voltage waveform distortion in the reference voltage $v_s(t)$ will introduce a "nonclean" $\cos(\omega t)$ signal into the circuit.

B. Proposed Circuit

To overcome the aforementioned problems, the block diagram of Fig. 2 has been modified by adding a feedback loop and an integral gain block, as shown in Fig. 3(a). To analyze the circuit, assume it has reached a steady-state condition. The hypothesis is that at the summing node, the inputs are $i_L(t)$ and $\bar{i}_p(t)$ (a precise estimation of the instantaneous active active current in the load). After the subtraction takes place, the feedback signal to the *active current estimator* block will be $i_F(t)$, as stated in (3). Then, by definition and in agreement with (3), $i_F(t)$ should not have an in-phase component because it has been subtracted from the load current $i_L(t)$. Otherwise, the input $\Delta\bar{I}_p$ in the integrator would not be zero, and then the circuit would not be in a steady-state condition previously assumed. After multiplication with the in-phase sinusoid, no dc component will be present, as can be observed in (4) by letting $I_p = 0$. This means that the integral gain block input $\Delta\bar{I}_p$, shown in Fig. 3(a) will be zero, which will keep constant the output of the integrator $\bar{I}_p$. Finally, if the output of the integrator corresponds exactly to the magnitude of the active current $\bar{i}_p(t)$, the input at the summing node will remain unchanged, and therefore the circuit will remain in this steady-state condition. In this form, the estimated active current $\bar{i}_p(t)$ is an accurate representation of the in-phase component in the power circuit.

Observe that scaling factors in the *analog multipliers*, in the *integral gain block* ($\int$), in the *low-pass filter*, or in the *in-phase sinusoidal generator* shown in Fig. 3(a), are not relevant to the circuit operation because of the closed-loop operation. Also, the estimated value of the active current $\bar{i}_p(t)$ could be used to measure the effective power applied to the load.

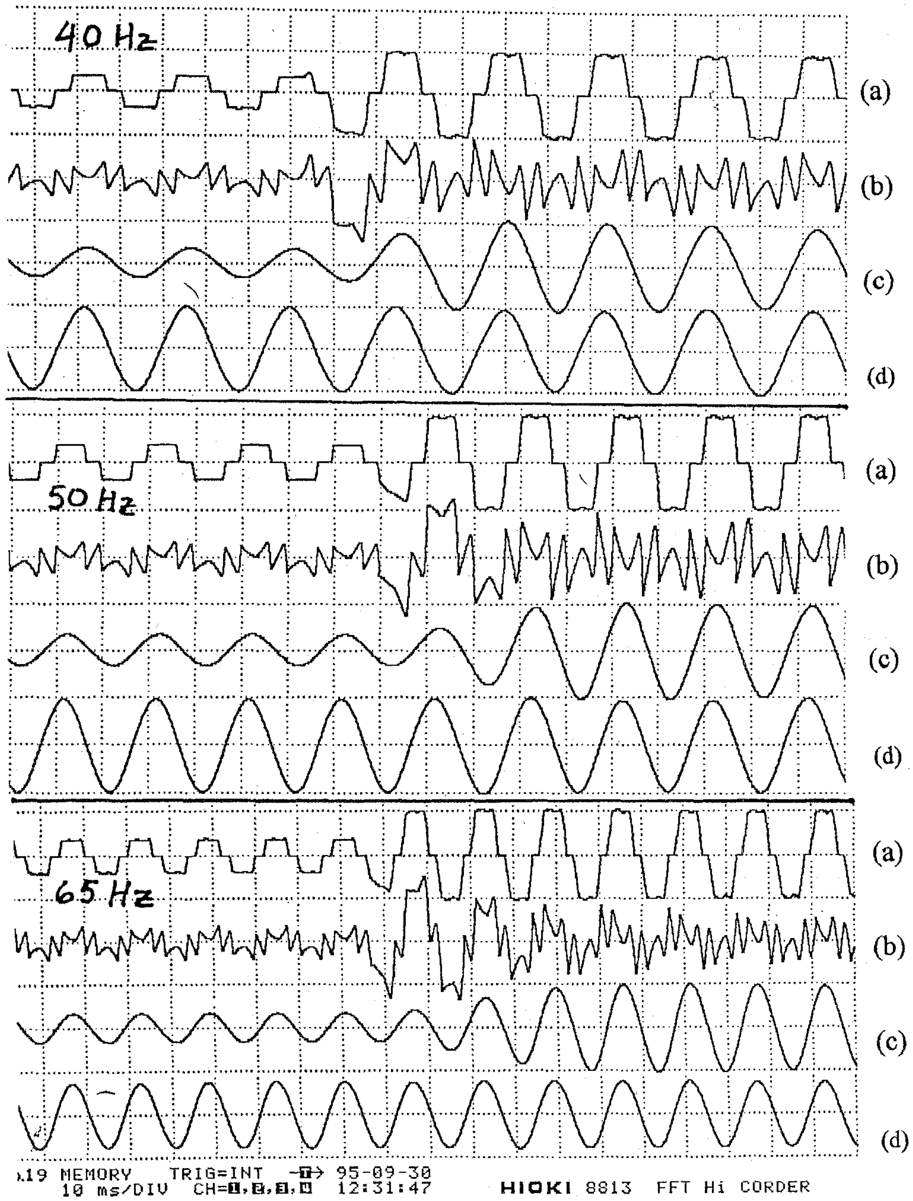

Fig. 6. Experiment with a rectifier load for 40, 50, and 65 Hz, respectively. (a) Load current $i_L(t)$, 10 A/div. (b) Calculated compensating current $i_F(t)$, 5 A/div. (c) Estimated active current $i_p(t)$, 10 A/div. (d) Line voltage reference $v_L(t)$, 5 V/div.

The presented method requires a low-distortion sinusoid with good phase tracking with respect to the line voltage. To get this clean in-phase sinusoid $[\cos(\omega t)]$, a *line voltage in-phase sinusoid generator* block is required. However, its amplitude is not relevant. It is not feasible to obtain this sinusoid by using a voltage transformer due to the relatively high third-harmonic present in most line utility voltages and the transformer phase shift. It is also impractical to apply a bandpass filter at the transformer output due to the frequency dependence of the output phase.

The method used to implement the *line voltage in-phase sinusoid generator* block shown in Fig. 3(a), consists in a sinusoidal PLL with error filtering and integration. This permits one to obtain zero phase error under steady state in a wide range of input frequencies, as shown in Fig. 3(b). As seen, the circuit output is a sinusoid that can follow input frequency variations over a wide range, restricted only by the control range of the sinusoidal voltage-controlled oscillator (VCO). The exact phase tracking with respect to the input is ensured by the integrator after the error filter. Since the PLL can accept a squarewave as input, the ideal transducer for this circuit is a fast optocoupler which not only provides galvanic isolation, but also ensures good phase tracking with respect to the line voltage.

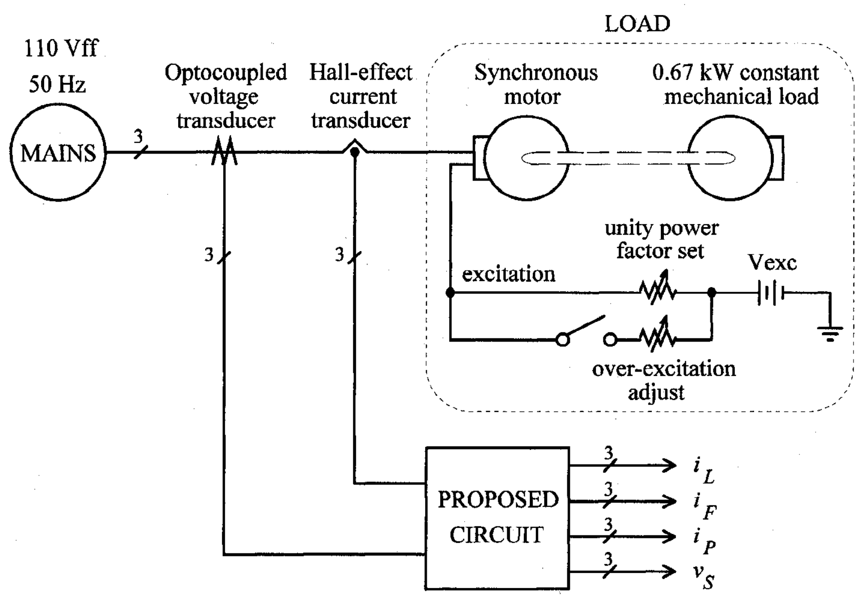

Fig. 7. Experimental circuit with a synchronous motor as a load.

C. Operational Characteristics of the Circuit

The following factors determine the dynamic behavior and stability of the proposed circuit:

1) closed-loop transfer function;
2) variations on circuit parameters (multiplier scale factors, in-phase sinusoid amplitude);
3) frequency response of the low-pass filter, which also affects the accuracy of i_F.

The resulting closed-loop transfer function determines basically the dynamic response of the circuit. Fig. 3(c) shows a simplified closed-loop diagram of the circuit suitable for analyzing its dynamic performance. For simplicity, a first-order low-pass filter has been chosen. The gain α accounts for scaling factors in the multipliers and variations in the in-phase sinusoid voltage. The open-loop transfer gain of the circuit $G(s)$ is shown in (5). The closed-loop transfer function $H(s)$, after rearranging terms, is shown in (6). It has a second-order polynomial with only positive coefficients in the denominator; therefore, it is always stable [7]

$$G(s) = \frac{\alpha^2}{s \cdot T_i \cdot (s \cdot T_o + 1)} \tag{5}$$

$$H(s) = \frac{\frac{\alpha^2}{T_i \cdot T_o}}{s^2 + s \cdot \frac{1}{T_o} + \frac{\alpha^2}{T_i \cdot T_o}}. \tag{6}$$

The damping characteristics of the denominator polynomial (overdamped, critically damped, or underdamped) determine the dynamic response of the circuit to a change in the input. It was found empirically that a slightly underdamped polynomial gives a good compromise between output overshoot and settling time.

The filtering characteristics of the open loop transfer function $G(s)$ in (5) affect the steady-state response of the circuit or, in other words, its accuracy. Equation (4) shows that the estimation of I_p is based on filtering the non-dc terms that appear. The lowest frequency is ω (the mains frequency), so in order to get a good estimation of I_p, $G(s)$ must provide enough attenuation for ω and above frequencies. Any frequency-dependent terms remaining after the filtering will introduce harmonic distortion on the estimated active current $i_p(t)$ and in the calculated compensating current i_F.

Finally, it can be seen in (6) that the variations in some circuit parameters (multiplier scale factors, in-phase sinusoid amplitude), represented by changes in α, modify the closed-loop transfer function, and therefore may modify the dynamic response of the circuit; however, it is likely to expect small variations only. Therefore, the dynamic performance will not change significantly. On the other hand, the steady-state response or the accuracy of the circuit is not affected.

The performance analysis of the in-phase sinusoid generator is less critical than the active current estimator because step changes in the mains frequency are not expected to occur. Care must be taken, however, to ensure that the sinusoidal PLL is capable of staying locked in the range of frequencies of interest.

III. Simulations and Experimental Results

A. Parameter Election

The parameters for the filter and integrator [see Fig. 3(c)] were adjusted empirically by means of computer simulations for a 50-Hz system. The time constant for the low-pass filter was adjusted to $T_o = 1.5\text{E}{-2}$ s. For the integrator $T_i = 1.5\text{E}{-3}$ s was used. The gain factor α of the multiplier and the in-phase sinusoid is 0.5 V/V (in fact, the multiplier introduces a 0.1-V^{-1} scaling factor and the in-phase reference sinusoid has 5-V amplitude, thus giving an overall gain factor of 0.5 V/V). According to these values and from (6), the closed-loop transfer function is

$$H(s) = \frac{11\,111}{s^2 + s \cdot 66.7 + 11\,111} = \frac{\omega_n^2}{s^2 + s \cdot \xi \cdot \omega_n + \omega_n^2}. \tag{7}$$

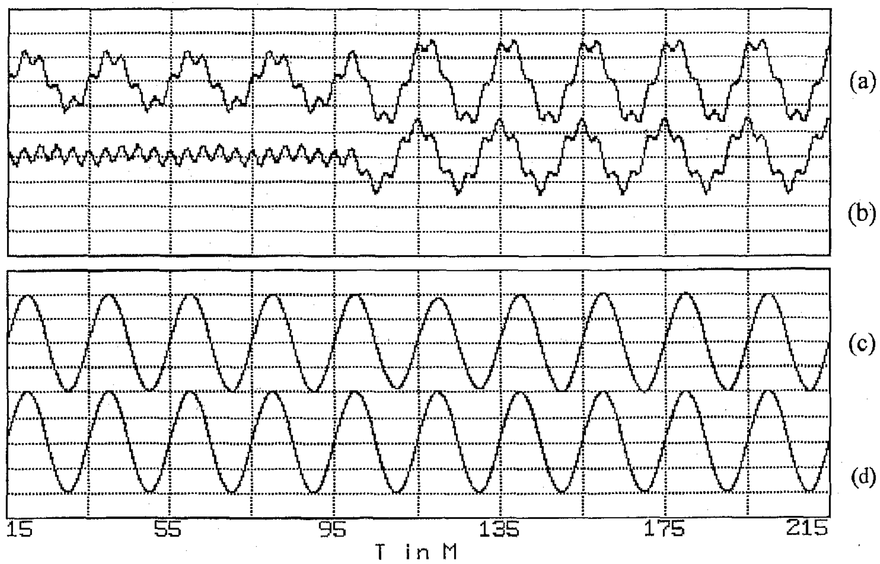

Fig. 8. Simulation for an excitation step in the synchronous motor (50 Hz). (a) Load current $i_L(t)$, 5 A/div. (b) Calculated compensating current $i_F(t)$, 5 A/div. (c) Estimated active current $i_p(t)$, 2.5 A/div. (d) Line voltage reference $v_L(t)$, 5 V/div.

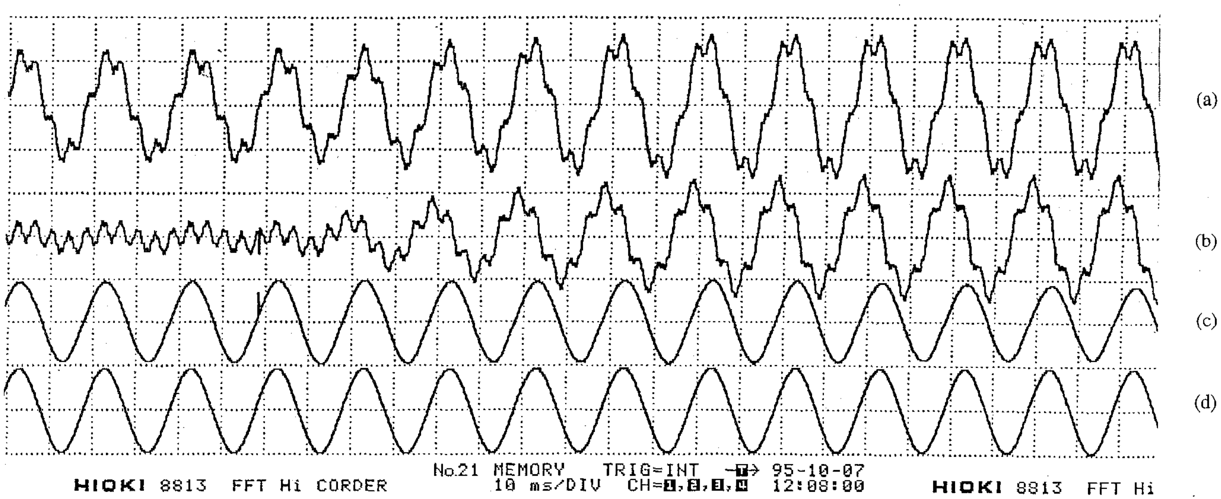

Fig. 9. Experiment for an excitation step in the synchronous motor (50 Hz). (a) Load current $i_L(t)$, 5 A/div. (b) Calculated compensating current $i_F(t)$, 5 A/div. (c) Estimated active current $i_p(t)$, 5 A/div. (d) Line voltage reference $v_L(t)$, 5 V/div.

In (7), ω_n is the natural resonance frequency and ξ is the damping factor. The resonance frequency is 105.4 rad/seg or 16.8 Hz. The damping factor is 0.633 (a value of two is a critically damped system). Both the natural resonance frequency and the damping factor are related to the response time of the circuit.

It is important to evaluate the overall attenuation for the undesired frequency-dependent terms shown in (4). The lowest frequency term that appears is dependent on ω (the mains frequency). The attenuation at this frequency is obtained by letting $s = j \cdot 2\pi \cdot (50\text{ Hz})$ in the open-loop transfer function $G(s)$ in (5), and then taking the absolute value. The result is 0.11 or -19.2 dB. At first sight this could be interpreted as a poor attenuation and a 11% undesired residue from this frequency. However, after inspecting (2) and (4), one comes to the conclusion that only the dc component and a second harmonic present in the load current can generate an ω frequency term, so they are likely to be small or not present at all. Further inspection of (4) shows that the next term is 2ω, given by the active and reactive current in the load. The attenuation obtained at this frequency is given by letting $s = j \cdot 2\pi \cdot (100\text{ Hz})$ in (5) and taking into account the additional 1/2 factor in (4). This gives an attenuation of 0.014 or -37.1 dB. From this result it is reasonable to suppose a near 2% harmonic distortion in the output.

The sinusoidal PLL was adjusted in order to operate properly in the range of frequencies from 40 to 65 Hz, however, a larger frequency operating range is possible.

B. Simulations and Experiments with a Rectifier Load

Several computer simulations and experiments were run with a rectifier load in order to check the performance of the circuit. Fig. 4 shows the circuit implemented for the experiment.

Fig. 5 shows a simulation with a load current similar to that of a six-pulse rectifier. Trace (a) is the load current $i_L(t)$. Trace (b) is the calculated compensating current $i_F(t)$. Trace (c) is the estimated active current $\bar{i}_p(t)$, which reaches a steady state after two and a half cycles. Finally, trace (d) is the line voltage reference $v_L(t)$.

Fig. 6 shows the experimental results obtained with the circuit shown in Fig. 4. The traces shown are the same as in Fig. 5, and the experiment was repeated for 40, 50, and 65 Hz without any adjustment (the circuit parameters used are the same as in Section III-A for the three frequencies). Again, it can be noted that the setup time is near two and a half cycles.

C. Simulations and Experiments with a Synchronous Motor

Fig. 7 shows the circuit implemented with a synchronous motor driving a constant mechanical load. The machine generates several harmonics due to its slots and the power factor can be adjusted by varying the excitation current.

With the machine driving a constant load in order to have a fixed active current, the excitation was adjusted until the power factor was near unity. Under these conditions, the compensating current $i_F(t)$ should correspond mainly to the induced harmonics since there is no reactive current. Then, the machine excitation was suddenly increased in order to have a reactive current step in the machine. After this, the estimated active current remains unchanged, as shown in the simulation of Fig. 8 and in the experiment of Fig. 9.

In Figs. 8 and 9, trace (a) is the machine current $i_L(t)$, trace (b) is the calculated compensating current $i_F(t)$, trace (c) is the calculated active current $i_p(t)$, and trace (d) is the reference line voltage. It should be noted that trace (c) remains constant in both the simulation and the experiment, and no significant perturbation is introduced by the reactive current step in the mains.

IV. CONCLUSION

A simple and effective method for calculating the current reference required by a shunt active power filter was presented. This method allows harmonic elimination and power factor correction in nonlinear loads.

It was experimentally verified that the proposed method requires no adjustment at all and it can tolerate circuit parameter variations. The circuit operated satisfactorily in the range of frequencies from 40 to 65 Hz without adjustments. However, a wider range can be obtained by appropriate parameter selection of the circuit components. These characteristics, along with its simplicity, demonstrate that this circuit is superior to traditional methods of evaluating the harmonic and reactive currents in a nonlinear load.

Simulations and experimental results showed that a transient response of two and a half cycles of the mains fundamental was achieved. In fact, this apparently slow transient response can be advantageous to soften transient phenomena from the mains point of view.

This method also calculates the active current in the load, which can be further used to measure the active power in the load.

Finally, the ability to operate under varying frequencies makes this circuit useful for variable frequency power systems.

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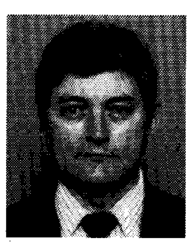

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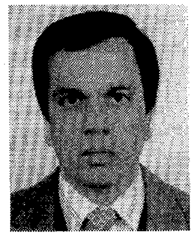

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